

# Bridge assessment loading: a comparison of West and Central/East Europe

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An objective of the European Commission 5th Framework Research project, Samaris, is to determine the bridge repair needs of new and possible future member states of the European Union (EU). This paper reports the findings of a study into the differences in bridge traffic loading between such countries and long-standing EU member states. A comparison is made of bridge traffic loading for two-lane short- to medium-length bridges in two countries deemed to be representative: Slovenia and the Netherlands. Truck classification and weight data from weigh-in-motion sites in each country are used in conjunction with other European data in a wide range of simulations to assess the implications of the differences for bridge assessments. Significant differences are identified between countries, both in the truck volumes and in the statistical distributions of truck weight. The implications for a range of bridge load effects are calculated and compared to the characteristic load effects implied by the Eurocode. Country- or network-specific bridge assessment load models are recommended to reflect the significant differences in traffic between European regions.

Keywords: Bridge; Load; Assessment; Traffic; Simulation; Statistics; Extreme value

#### 1. Introduction

Bridge repair needs vary considerably across Europe for two reasons: the current state of repair of bridges is different in the various national road networks due to differences in traditions of bridge assessment and repair; and the traffic loading on bridges is different due to differences in traffic volume and the statistical distributions of Gross Vehicle Weight (GVW) (OBrien et al. 2005). Differences in the GVW distribution arise from the current state of economic development and the regulatory and enforcement environment in a country. This paper reports typical variations in bridge repair needs that arise from variations in the loading due to heavy truck traffic.

Increasingly, truck weight statistics are being used to determine bridge design and assessment loading (Dorton and Csagoly 1977, Cooper 1995, O'Connor et al. 2001). The truck volume and statistical distribution of truck

weights clearly has a significant influence on the bridge repair needs of a road network. In a region with a high level of manufacturing for example, there will be greater volumes of heavy trucks. In bi-directional traffic, higher truck volumes increases the probability of two or more heavy trucks meeting on a bridge, thereby increasing the characteristic load effects (such as bending moment and shear force). It can be argued that truck volumes in the new member states of the European Union will eventually reach those levels experienced in the most developed regions. This is a valid argument in the context of new bridge design where the marginal cost of providing additional capacity is small and where the bridge design life is 100 years or more. However, for existing bridges it is not practical, nor is it a wise use of resources, to upgrade all bridges instantly. Bridge repair and rehabilitation is an ongoing process and an understanding of *current* characteristic load effects provides a means to prioritise projects and optimise the

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use of limited funds. There will also be many cases where bridges designed for low traffic loading may continue to function safely in the long term, due to low levels of heavy goods vehicle traffic in the region. In this paper the differences in heavy goods vehicle traffic between a new and a long-standing EU member state are considered along with the implications of those differences for characteristic load effects and hence for bridge repair needs.

There are two key ways in which traffic loading can vary: truck flow (volume) and the statistical distribution of truck GVWs. Higher flow clearly has an influence on characteristic load effect. However, the shape of the GVW histogram is also important, and can vary significantly as illustrated in figure 1. For example, when there is strong overload enforcement activity, there may be large numbers of trucks (and hence a sharp peak in the histogram) at or near the legal weight limit. On the other hand, when there is less enforcement or a greater number of permits for heavy trucks, the truck weights near the conventional legal limit may have a greater variance tending towards the high GVW side.

For this study, traffic data was analysed from Weigh-In-Motion (WIM) stations in the Netherlands (NL), and in Slovenia (SI). The NL sites are assumed to be representative of traffic in a long-standing EU state which is heavily industrialised. The SI sites are assumed to be representative of traffic in a new EU member state with a growing economy.

The GVW histograms of the French and Polish sites illustrated in figure 1 are presented for comparison. There are similarities between the Dutch and Polish sites. Both have large numbers of very light vehicles, suggesting mixed use and a high percentage of vans or large passenger vehicles. Both also have a peak below 20 tonnes, suggesting a significant number of loaded 2- or 3-axle trucks. Such vehicles would be typical of local industrial activity, rather than international haulage. Both also have a modest frequency of vehicles around 40 tonnes, representative of a typical 5-axle loaded truck. It is noteworthy however that the tail of the distribution for the Dutch site extends considerably further, approaching 60 tonnes. The tail of the Polish distribution on the other hand ends at a little above 50 tonnes. This may reflect the legal limits (without special permits) of 50 tonnes and 44 tonnes for the Netherlands and Poland respectively. The difference is highly significant in the context of bridge loading.

There are also similarities in the shapes of the French and Slovenian sites. Both have more uniform distributions than the Dutch and Polish sites. Both have a relatively low percentage of light vehicles and a lot of mid-range truck



Figure 1. Sample GVW histograms from four European countries: (a) Site 2 in the Netherlands; (b) Site 2 in Slovenia; (c) French site  $- A5$ ; (d) Polish site.

weights, probably a mixture of unloaded 4- and 5-axle trucks and loaded 2- and 3-axle trucks. However, there are significant differences in the tails. In Slovenia, there is a high percentage of trucks close to 40 tonnes, suggesting good compliance with the legal limit of 44 tonnes. The tail ends abruptly at about 45 tonnes. In France on the other hand, there are significant numbers of trucks at 50 tonnes and the tail extends towards 60 tonnes. In this key feature, the French and Dutch sites are quite similar – both of these tails extend considerably higher than the Slovenian and Polish sites.

#### 2. Notional load model ratios

Six weigh-in-motion sites were selected for the study, Gravendeel (NL Site 1), Woerden (NL Site 2) and Hoofddorp (NL Site 3) in the Netherlands and Crnivec (SI Site 1), Postojna (SI Site 2) and Trojane (SI Site 3) in Slovenia. The sites involve different numbers of lanes per carriageway. For example, SI Site 2 data is from the slow lanes of a 4-lane motorway. As lighter vehicles tend to travel in the fast lane, the resulting histogram has a higher density of heavy vehicles than would be the case on a 2-lane bridge. It is conservative to use slow lane WIM data from a multi-lane carriageway to assess load on bridges with fewer lanes. However, in the absence of more appropriate sites, such a practice is not uncommon (O'Connor *et al.* 2001).

Clear differences are found in the flows and the shapes of GVW histograms between sites in the two countries. To quantify the implications of the differences, three characteristic load effects are calculated for three bridge lengths using statistical extrapolation methods. In order to provide a basis for comparison, all characteristic load effects are expressed as a ratio of the value found using a standard notional load model. The Eurocode Normal load model (EC 1, 1994) for an 8 m wide bridge with two notional lanes (figure 2) is used for this purpose. Results from this model are divided by dynamic factors (Bruls et al. 1996) to

provide static load effects such as those found from the simulations. The characteristic load effect values calculated from the simulations, divided by the corresponding static value found using the notional model, is referred to here as the Notional Load Model Ratio (NLMR).

For the three Slovenian and the three Dutch WIM sites, data consisted of:

- hourly flows for each direction;
- . composition of the truck traffic based on the number of axles for each direction;
- . GVW distribution for each direction and number of axles.

There were very few (less than  $2\%$ ) trucks with more than 5 axles and, for simulation purposes, these trucks were ignored, an assumption found to be reasonable in another study (Gretchew and O'Brien 2005). Other required information was taken from WIM data obtained at various sites in France (Grave et al. 2001), including:

- speed, by direction;
- . axle spacings for each direction and class of vehicle (number of axles);
- . axle weight distributions as a function of GVW for each class and direction.

The above sets of data were used to obtain a single notional European site in all but flow and GVW characteristics which were retained for each site. This arrangement allowed the examination of the effect of differing GVW distributions and flows on the traffic loading while keeping all other parameters constant.

A number of analyses were carried out with a notional flow of 7500 trucks per day to determine the influence of histogram shape on the results. Other analyses were carried out to determine the effect of flow. Low- and high-flow extremes represented by 20% and 200% of the notional



Figure 2. Eurocode Normal traffic loading (alpha-factors all taken as unity).

were compared. 250 working days were simulated, effectively 1 year of traffic, in all cases except for the 200% flow. In these cases, 50 days were simulated due to computingpower limitations. This will not have a significant effect on the characteristic value as the results exhibited extreme value population characteristics.

The simulations were carried out for 3 bridge lengths: 15, 25 and 35 m and for 3 load effects: bending moment at the centre of a simply supported span, hogging bending moment over the central support of a 2-span bridge and shear at the left hand support of a simply supported beam (labelled 1, 2 and 3 respectively). For Load Effect 2, the length is the total length of both spans of the bridge. For the bridge length of 15 m, two 7.5 m spans represents an unrealistic bridge configuration and hence no results are presented for this length/load effect combination.

The GVW histograms are initially fitted with a trimodal Gaussian distribution (a composite distribution consisting of three weighted Normal distributions). However, this is subsequently adjusted using a log scale to improve the fit to the important right-hand tail. This has the effect of de-emphasising the quality of fit in the main body of the distribution in order to improve the accuracy in the right-hand tail region. Axle spacings are modelled as uni- or bi-modal Normal distributions. Axle weights for 2- and 3-axle trucks are modelled as tri- or bi-modal Normal distributions, whereas those for 4- and 5-axle trucks are modelled as Normal distributions with axle weight expressed as a percentage of GVW for the first and second axles and for the remaining tandem/tridem group. The measured percentages of 2-, 3-, 4- and 5-axle trucks are used to determine the traffic composition. The average flow rate for each hour of the day is determined from the flow rates for that hour across all the working days of the measurement period. Headways are generated using the normalised headway method of Crespo-Minguillón and Casas (1997). A checking procedure is used to ensure that a minimum gap of 5 m is maintained between the last axle of the leading truck and the front axle of the following truck (Harman and Davenport 1979, Bailey 1996, Grave 2001). An alternative more elaborate

headway model is described by OBrien and Caprani (2005). Speed is modelled as a Normal distribution and is considered independent of truck type.

Based upon the statistical distributions fitted to the measured data, a Monte-Carlo procedure was used to generate artificial streams of truck traffic whose characteristics match those measured, yet are allowed vary from the actual measured values. The generated truck traffic streams for each site were simulated crossing the bridge. To minimise processing requirements, only 'Significant Crossing Events' (SCEs) are considered. An SCE is defined as any multiple-truck presence event on the length considered or the presence of a single truck with GVW in excess of 40 tonnes. Crossings of lighter individual trucks were found not to result in maximum-per-day load effects. When an SCE is identified, the comprising truck(s) are moved in 0.02 second intervals across the bridge and the maximum load effects for the event are retained for further analysis. Typical proportions of different event-types are given in table 1. In this table, the event-type is known by the maximum number of trucks concurrently present on the bridge at any point between periods in which there are no trucks present.

The maximum load effect for each SCE is a random variable; the collection of these maximum load effects forms a parent distribution and the greatest load effect value for all SCEs that occur in a day (daily maximum) can be fitted to an Extreme Value distribution (Fisher and Tippett 1928, Gumbel 1958, Ang and Tang 1975, Coles 2001). In bridge traffic loading literature, Extreme Value distributions are regularly used to extrapolate from the recording period to the return period. Flint and Jacob (1996) provide several methods that were compared for the Eurocode 1, Part 3 studies. Much statistical research has been directed in the area of Extreme Value distributions (Galambos 1978, Castillo 1988) and most authors utilise this form of analysis (Bailey and Bez 1994, OBrien et al. 1995, Grave et al. 2000, Caprani et al. 2002, Moyo et al. 2003). A detailed review of the statistical analysis of bridge load effects has been presented by Caprani (2005).

		Event Type						
Country	Bridge Length	1-truck $(>40t)$	2-truck	3-truck	4-truck			
Netherlands (NL)	15 m	0.336	0.637	0.02732	0.00045			
	$25 \text{ m}$	0.261	0.684	0.05299	0.00216			
	35 m	0.214	0.703	0.07769	0.00485			
Slovenia (SI)	15 m	0.236	0.734	0.02939	0.00047			
	$25 \text{ m}$	0.176	0.764	0.05777	0.00219			
	35 <sub>m</sub>	0.140	0.772	0.08291	0.00530			

Table 1. Average proportion of different event-types by country and bridge length.

In this study, the daily maxima are fitted to a Gumbel distribution to obtain the design load effect. This is defined in Eurocode 1, Part 3 as that load effect which has a 10% probability of excedance in 100 years, giving a 1000-year return period. The characteristic load value for this return period is determined using the fitted Gumbel distributions. As the daily flows were usually quite large, ensuring a large parent population, the daily maxima generally fit well to the Gumbel distribution. In the cases of low flow (20% of notional), the asymptotic convergence is less good and, as recommended by Castillo (1988), the distribution was fit to the  $2\sqrt{n}$  greatest of the maxima where *n* is the number of maxima used.

### 3. Effect of GVW histogram shape

There are significant differences in the shapes of the histograms between the sites considered as can be seen in the typical examples illustrated in figure 1. The shapes of the histograms for the French and Dutch sites in this figure are quite different for lower weights. However, they both have a peak around 43 tonnes and a right tail which extends to around 55 to 60 tonnes. As the French peak is higher, its behaviour will be similar to the Dutch site with a higher truck volume. The Slovenian site is noteworthy for the sharp peak at about 40 tonnes and a sudden drop off at around 45 tonnes. The Polish site is different again with a very flat peak around 40 tonnes and a tail extending to about 50 tonnes. Only the Dutch and Slovenian histograms are considered further in this paper.

To identify the influence of the shape of the histogram of GVW, a notional truck flow was used for all sites. This notional flow was defined as 3423 trucks per day in Direction 1 and 4077 in Direction 2. The effect of histogram shape is presented in table 2 and figure 3. For all load effects and lengths except one (35 m bridge length, Load Effect 2), all three SI sites have NLMRs less than all three

NL sites. This is the result of a greater proportion of NL than SI trucks being in the right-hand (heavy) end of the histogram.

The averages for the three sites in each country are also presented in table 2 as well as the difference in the averages expressed as a percentage of the NL average. It can be seen that the average NLMRs in the SI sites are between 6.5% and 18.6% less than the corresponding Dutch averages. Given that the same flow and other traffic characteristics are used in both cases, this is a considerable difference which results purely from the shape of the histograms.

#### 4. Effect of flow

There are great differences in the flows between the NL and SI sites as can be seen in tables 3 and 4 and figure 4. Both the total numbers of trucks and the key numbers of 5-axle trucks are significantly different between the sites in the two countries. It can be seen in table 4 that the average of the three SI sites has only 36% of the average daily recorded truck numbers at the three NL sites. Furthermore, the SI sites have only 43% of the 5-axle trucks of the NL sites and it is only in the 2-axle truck category that the SI sites exceed half the flow of the NL sites.

To identify the influence of flow, 'typical' shapes of histogram are defined for the SI and the NL sites. These are determined by calculating the average, for the three sites of the given country, of the normalised frequency for each weight interval. Average national histograms are determined in this way for each direction and vehicle type (number of axles).

For the two notional national sites, two new flows were considered, 20% notional and 200% notional. Hence, this flow study compares a total flow of 1500 to a total flow of 15 000. The results are given in table 5 and illustrated in figure 5. For the notional NL histogram, flow has a significant effect. As would be expected, increasing the flow ten-fold significantly increases the NLMRs for all lengths

	15 <sub>m</sub>		25 <sub>m</sub>			35 <sub>m</sub>		
	Effect 1	Effect 3	Effect 1	Effect 2	Effect 3	Effect 1	Effect 2	Effect 3
Netherlands (NL)								
Site 1	0.69	0.72	0.72	0.76	0.73	0.68	0.68	0.69
Site 2	0.68	0.70	0.70	0.73	0.71	0.65	0.71	0.69
Site 3	0.64	0.71	0.66	0.71	0.70	0.67	0.78	0.72
Mean	0.67	0.71	0.69	0.74	0.71	0.67	0.72	0.70
Slovenia (SI)								
Site 1	0.55	0.59	0.57	0.62	0.59	0.55	0.64	0.57
Site 2	0.54	0.58	0.55	0.63	0.59	0.58	0.70	0.64
Site 3	0.55	0.60	0.58	0.66	0.64	0.60	0.68	0.60
Mean	0.55	0.59	0.57	0.64	0.61	0.57	0.68	0.61
Difference	$18.6\%$	$16.3\%$	$18.1\%$	13.3%	$15.2\%$	13.8%	$6.5\%$	$13.0\%$

Table 2. Notional Load Model Ratios.



Figure 3. NLMRs for Dutch (NL) and Slovenian (SI) sites: (a) Load Effect 1; (b) Load Effect 2; (c) Load Effect 3.

Table 3. Daily flow of trucks at 6 sites by axle number.

		Netherlands		Slovenia			
	NL Site 1	NL Site 2	NL Site 3	SI Site 1	SI Site 2	SI Site 3	
$2$ -axle	2595	2460	1950	1076	1217	1692	
$3$ -axle	1107	1147	961	209	257	297	
4-axle	2392	2652	3412	177	633	344	
5-axle	2896	3730	7031	645	3255	1972	
Total	8990	9989	13354	2107	5362	4305	

Table 4. Average daily flows of trucks at SI and NL sites.



and load effects. Similarly for the SI histogram, increasing the flow rate ten-fold significantly increases the ratios. It is interesting to see the relative sensitivity of the ratios to the flow rate. The increase in flow rate in SI generally has less

effect than in NL. This is likely related to the higher numbers of heavy trucks in the latter which results in an exponentially increasing number of extreme meeting events as flow increases. The effect of increasing the flow ten-fold in NL is of a similar order of magnitude to the difference between low flow in SI and NL. This means that, in determining NLMRs, the shape of the histogram is approximately as important as a ten-fold increase in flow.

### 5. Combined effect of GVW histogram and flow

The total effect of different truck volumes and different histograms in all six sites is presented in table 6. All three SI



Figure 4. Total number and number of 5-axle trucks per day at 3 NL and 3 SI sites.

		15 <sub>m</sub>		$25 \text{ m}$			35 <sub>m</sub>		
	Effect 1	Effect 3	Effect 1	Effect 2	Effect 3	Effect 1	Effect 2	Effect 3	
Netherlands (NL)									
$0.2 \times$ Flow	0.55	0.65	0.64	0.65	0.57	0.57	0.49	0.60	
$2 \times$ Flow	0.72	0.74	0.72	0.80	0.75	0.69	0.83	0.75	
Slovenia (SI)									
$0.2 \times$ Flow	0.49	0.58	0.56	0.59	0.53	0.50	0.50	0.51	
$2 \times$ Flow	0.58	0.60	0.58	0.66	0.63	0.60	0.75	0.65	

Table 5. NLMRs for notional national histograms.

sites have NLMRs that are consistently less than all three NL sites. The mean difference ranges from 16.2% to 25.0%. This is due to the combined effect of greater flows in NL are combined with GVW distributions with a greater number of heavy trucks. These are significant differences and suggest a considerably less onerous load model for the assessment of bridges near the Slovenian sites.

## 6. Discussion and conclusions

The effect of loading on the characteristic values for three common load effects is considered in this paper. It is shown that there are great differences in the flows and GVW histograms between NL and SI sites. This can be seen to have a most significant effect on the characteristic values and hence on the NLMRs. Hence, for a given bridge capacity, there is a much greater safety margin in SI than in NL due to the lower level of traffic loading implicit in the WIM records. Bridges throughout Europe are assessed using a range of techniques and it is typical to assess a bridge for a notional load model. This study has shown that a considerably less onerous model is appropriate for 2-lane

bridges in SI at this time than for NL. When a bridge is strengthened or replaced, then it should, in the authors' opinions, be designed for full Eurocode loading which allows for future traffic growth. However, there are many bridges in the new member states which can function safely without being strengthened or replaced because the traffic loading is considerably less than in countries such as NL. This is a very significant finding as it can prevent a great deal of unnecessary strengthening and replacement of bridges.

Considerable resources will be required to upgrade the transport infrastructure in the new EU member states. These resources can be used to much greater effect if account is taken of traffic loading and its implications for bridge repair needs. This can be done through countryor region-specific notional traffic load models that reflect existing levels of heavy goods vehicle traffic rather than those appropriate to more heavily trafficked regions. It is not possible to accurately quantify the potential savings as the link between different levels of loading and rehabilitation requirements is difficult to determine. Nevertheless, it is apparent that great savings may be made.



Figure 5. NLMRs for different flows: (a) Load Effect 1; (b) Load Effect 2; (c) Load Effect 3.

	15 <sub>m</sub>		25 <sub>m</sub>			35 <sub>m</sub>		
	Effect 1	Effect 3	Effect 1	Effect 2	Effect 3	Effect 1	Effect 2	Effect 3
Netherlands (NL)								
Site 1	0.69	0.74	0.73	0.76	0.76	0.70	0.72	0.71
Site 2	0.70	0.72	0.73	0.78	0.72	0.68	0.76	0.72
Site 3	0.63	0.69	0.69	0.76	0.73	0.67	0.75	0.72
Mean	0.67	0.72	0.72	0.77	0.74	0.68	0.74	0.72
Slovenia (SI)								
Site 1	0.53	0.54	0.51	0.58	0.54	0.51	0.55	0.51
Site 2	0.54	0.58	0.54	0.62	0.59	0.56	0.67	0.60
Site 3	0.52	0.59	0.55	0.62	0.61	0.56	0.64	0.57
Mean	0.53	0.57	0.54	0.61	0.58	0.54	0.62	0.56
	$20.9\%$	$20.8\%$	$25.0\%$	20.8%	$21.6\%$	$20.6\%$	$16.2\%$	$22.2\%$

Table 6. NLMRs for site-specific GVW distributions and flows.

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