Estimating Extreme Highway Bridge Traffic Load Effects

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ABSTRACT: This study applies the Box-Cox-GEV statistical model to estimation of bridge traffic load effect. This model negates the need for choice between the peaks-over-threshold or block maxima extreme value theories. This model is further developed within the framework of a recently-presented method that allows for the distributions of load effect resulting from different loading event types. This combined novel approach is applied to weigh-in-motion-derived traffic for a range of bridge lengths and load effects. The sensitivity of load effect prediction to various choices is assessed in comparison to current state-of-the-art. It is shown that the standard extreme value theories are strongly rejected in favor of the new model, through use of likelihood ratio testing. Further, an optimum threshold for extrapolation is found, along with a suitable range of the model parameter. Lastly, it is shown that this new approach gives slightly higher lifetime load effect values than other competing approaches.

1 INTRODUCTION

1.1 *General*

In the reliability analysis of a bridge structure, traffic induced load effects are some of the most variable parameters. Because of this, more accurate estimation of the distributions of traffic load effect can result in a significant improvement in the accuracy of calculated safety levels.

For the short- to medium-span bridges considered in this study, free-flowing traffic, including dynamic interaction between the vehicles and bridge, generally governs extreme load effect. In assessing such loading, it is common to measure the traffic at the site, parameterize the statistical distributions of traffic characteristics, and use Monte Carlo simulation to artificially extend the measurements, since they are expensive to obtain. Typically, this data forms the basis for a subsequent extreme value analysis.

This paper provides a means to improve the extreme value analysis though use of a more general extreme value distribution. It does so, within the framework of a recently proposed method which accounts for the different distribution of load effect caused by different types of loading event.

1.2 *Extreme Value Theory*

There are two main methods of extreme value theory (EVT) (Coles 2001). The block maxima approach uses the maximum of the data obtained for a block (a period of time). The block must be longer than the period of any underlying variation in the statistical process, such as hourly traffic flow rates. Many such blocks give a population of maxima to which the Generalized Extreme Value (GEV) distribution (incorporating the Fisher-Tippett families) is applied.

The block maxima approach is wasteful of data; measurements may be taken throughout the period and yet only lead to a single data point: the maximum. Also, the second highest value in one block may be larger than the highest of another block and this is not accounted for generally.

The Peaks Over Threshold (POT) approach avoids some of the problems of the block maxima method through use of the Generalized Pareto Distribution (GPD). However, it involves the choice of a threshold on which the results depend.

Importantly, the choice of EVT method is usually subjective whilst the results of the two methods are generally different. Recently, the Box-Cox-GEV model has been proposed by Bali (2003). This model encompasses both the GEV and GPD distributions and thus the two main approaches of EVT. Therefore, in applying this model, the data itself determines the most appropriate form of EVT analysis.

1.3 *Bridge Traffic Load Effect Prediction*

In the literature on bridge traffic load effect estimation, load effects have been found from directly-measured traffic; Monte Carlo simulated traffic, and;

convoluted traffic. It is the methods of extrapolating this load effect data that is of interest here.

In the background studies for the development of the Eurocode for bridge loading (EC1.2 2003), Bruls et al. (1996) and Flint & Jacob (1996) consider various methods of extrapolation, including:
- a half-normal curve fitted to the histogram tail;
- a Gumbel distribution fit to the histogram tail;
- Rice's formula for a stationary Gaussian process;

Other authors usually consider only one approach.

1.3.1 *Use of Parent Distributions*

In the papers Nowak (1989) and Nowak & Hong (1991), straight lines are fit to the tails of the load effect distributions plotted on normal probability paper. Instead, Nowak uses curved lines to extrapolate for the load effects of various return periods in Nowak (1993) and Nowak (1994).

Rice's formula has been used extensively in the literature (Flint & Jacob 1996, Cremona 2001). This method involves the choice of a threshold; Cremona (2001) develops an optimal level at which to set the threshold, based on minimization of the Kolmogorov-Smirnov statistic.

1.3.2 *Use of Distribution of Maximum*

Nowak (1993) determines the distribution of maximum load effect by raising the parent distribution of load effect to an appropriate power. In this way he determines the mean and coefficient of variation of the maximum load effect.

Fu & Hag-Elsafi (1995) describe a probabilistic convolution method to obtain bending moments for single truck events. They then obtain the distribution of maximum load effect by raising the parent distribution to an appropriate power.

Ghosn & Moses (1985) use a 2.4 hour maximum as their extreme data which is then fitted using a normal distribution on normal probability paper. This distribution is then raised to the appropriate power obtain the 50-year load effect distribution.

1.3.3 *Use of Extreme Value Theory*

Buckland et al. (1980) use a Gumbel distribution to fit 3-monthly maximum load effect which is then used to extrapolate to the return periods of interest. Similarly, Cooper (1995, 1997) raises the distribution of measured load effect to a power to get the 4.5-day distribution of maximum load effect. This is modeled with a Gumbel distribution, which is used to extrapolate to a 2400-year return period.

Bailey & Bez (1994 and 1999) determine that the Weibull distribution is most appropriate to model load effect tails and used maximum likelihood estimation. In Moyo et al. (2002), daily maximum bridge strain measurements are fit to a Gumbel distribution using least-squares on probability paper.

Lastly, but notably, Crespo-Minguillón & Casas (1997) adopt the POT approach and use the GPD to model the exceedances of weekly maximum traffic load effect over a threshold. An optimal threshold is selected based on the overall minimum least-squares value, and the distribution corresponding to this threshold is used as the basis for extrapolation.

1.3.4 *Summary*

From this brief review of the literature, it is apparent that there is a wide range of methods used in the estimation and prediction of load effect. Many are quite subjective and this influences the load effect predictions. However, the trend in recent years is towards the full implementation of EVT. Doing so removes much subjectivity lending further confidence to the resulting load effect predictions.

2 THEORETICAL BASIS

2.1 *Basis for Estimation*

The passage of vehicles over a bridge imparts load effect to the structure. We consider individual loading events as events that occur between periods of no truck-traffic on the bridge. During such events, a history of load effect is determined, the maximum value of which is retained for further analysis. It is usual to use influence lines for the calculation of load effect and so it is load effect at a particular location in the structure that is determined, which is not necessarily the maximum value occurring.

2.2 *Conventional Approach*

For later comparison we develop here a synthesis of the current state of the art, as previously discussed. We term this a generic 'conventional approach'. We consider that the population of load effect data relating to individual loading events is processed to find the absolute maximum value of load effect per day, for each load effect considered. Also, maximum likelihood estimation is used to fit a GEV distribution to this population of maxima. This synthesized approach removes many of the sources of subjectivity in the current literature, such as: choice of population; the extreme value distribution used; the means of estimation; and the choice of threshold.

2.3 *Composite Distribution Statistics*

An important assumption of classical EVT is that the statistical mechanism is independent and identically distributed (iid) – though some interdependence is tolerable (Galambos 1978). Caprani et al. (2008) have shown that the iid assumption is not valid for bridge loading events. In particular, as is intuitively reasonable, the distribution of load effect for two trucks concurrently present on the bridge (a 2-truck event) is quite different from that of three-trucks (a

3-truck event). Separating the loading event types, the exact distribution of load effect (from the Theorem of Total Probability) is:

$$P\left[S \le s\right] = \left(\sum_{i=1}^{N} F_i(s) \cdot f_i \right)^{n_d} \tag{1}$$

where S is load effect; $F_i(\cdot)$ is the cumulative distribution function (CDF) of loading event-type i which has frequency of occurrence f_i; there are n_d number of loading events per day, and; N loading event types to be considered. Caprani et al. (2008) show that this distribution asymptotically approaches a composite distribution statistics (CDS) model, $G_C(\cdot)$:

$$G_C\left(s\right) = \prod_{i=1}^{N} G_i\left(s\right) \tag{2}$$

where $G_i(\cdot)$ is any extreme value distribution. When the block maxima method is used, the GEV distribution for loading event type i is:

$$G_i\left(s\right) = \exp\left\{ -\left[1 - \xi_i \left(\frac{s-\mu_i}{\sigma_i} \right) \right]_+^{1/\xi_i} \right\} \tag{3}$$

where $[h]_+ = \max(h, 0)$ and the parameters, μ_i, σ_i, ξ_i, are found by fitting to the load effect data of loading event type i solely.

Typical use of the CDS method involves the daily maximum load effect for each loading event type, to which separate GEV fits are made. The final distribution of load effect then is found from Equation 2.

The CDS approach can be applied to any extreme value distribution that meets the stability postulate (Caprani et al. 2008).

2.4 *The Box-Cox-GEV Model*

Bali (2003) introduces the Box-Cox-GEV (BCGEV) extreme value distribution:

$$H\left(s\right) = \left(\frac{1}{\lambda}\right)\left(\left[\exp\left\{-\left[h\left(s\right)\right]_+^{1/\xi}\right\}\right]^{\lambda} - 1\right) + 1 \tag{4}$$

in which

$$h\left(s\right) = 1 - \xi\left(\frac{s-\mu}{\sigma}\right) \tag{5}$$

The parameters of this distribution are those of the GEV distribution (μ, σ, ξ) and a 'model parameter', λ. As $\lambda \to 1$, the BCGEV converges to the GEV distribution. Conversely, as $\lambda \to 0$, by L'Hopital's Rule, the BCGEV converges to the GPD. To apply this model a high threshold is set on the parent distribution. Bali (2003) uses a threshold of two standard deviations about the sample mean.

Bali & Theodossiou (2008) found that maximum likelihood estimation of the BCGEV parameter is not robust and this has been found to be the case for this work also. Therefore, estimation has been carried out using nonlinear regression, as proposed by Bali (2003). Arranging the data in increasing order, $s_1 \le \ldots s_r \le \ldots s_n$, we compare the expected value of $H(s)$ for each data point to its empirical value:

$$E\left[H\left(s\right)\right] = \frac{r}{n+1} \tag{6}$$

Substitution of Equation 4, followed by twice taking logarithms and by adding a residual (or error) term, η, we get the nonlinear regression equation:

$$\log\left[\left(-\frac{1}{\lambda}\right)\log\left(1+\lambda\left(\frac{r}{n+1}-1\right)\right)\right] = \frac{1}{\xi}\log\left[1-\xi\left(\frac{s_r-\mu}{\sigma}\right)\right] + \eta \tag{7}$$

By minimizing the sum of the squared residuals (SSR), $\Sigma\eta_i^2$, parameter estimates are obtained. Then, because the GEV and GPD models are nested in the BCGEV model, the likelihood ratio test can be used to determine, for a given significance level, whether the GEV or GPD model is appropriate.

2.5 *Likelihood Ratio Test*

Huet et al. (2003) describe the use of the likelihood ratio for nonlinear regression models. Using the standard error of regression (SER), defined as the mean SSR (SSR/n), the likelihood ratio (LR) statistic for nested models is given by:

$$\mathrm{LR} = n\left(\log \mathrm{SER}_P - \log \mathrm{SER}_F\right) \tag{8}$$

where the subscript P refers to the partial model (the GEV or GPD distribution) and F refers to the full model (BCGEV). This LR statistic is approximately χ^2-distributed with one degree of freedom.

The hypothesis that the considered partial model is adequate to describe the data is rejected if the LR statistic is greater than the critical value at a significance level of 1-α. In the following we consider the 5% and 1% significance levels, the critical values for which are 3.842 and 6.635.

3 APPLICATION TO BRIDGE TRAFFIC LOAD EFFECT

3.1 *Description of Study*

3.1.1 *Traffic Data Basis*

Five working days of weigh-in-motion data was taken from the A6 Paris to Lyon motorway near Auxerre, France. Truck traffic characteristics, such as weight and dimensional data, were collected for

36,373 trucks, travelling in the two slow lanes. These characteristics were statistically modeled (Caprani 2005) for use as the basis of Monte Carlo simulation. The model used for the distribution of headways is particularly important and is described by OBrien & Caprani (2005).

We consider bridges with only two opposing lanes of lengths in the range 20 to 50 m. The load effects examined are:

- Load Effect 1: Bending moment at the mid-span of a simply-supported bridge;
- Load Effect 2: Bending moment at the central support of a two-span continuous bridge;
- Load Effect 3: Left hand shear in a simply-supported bridge.

Monte Carlo simulation was used to generate a 1000-day period of truck traffic. This truck traffic was used to determine the load effect values for the bridges and load effects considered. Only significant crossing events, defined as multiple-truck presence events and single truck events where the vehicle's Gross Vehicle Weight (GVW) is in excess of 40 tonnes, were processed to minimize computing requirements. For such events, the comprising truck(s) are moved in 0.02 second intervals across the bridge and the maximum load effects of interest identified. The set of 1000 daily maximum load effect values for each loading event type were determined. This enables the direct application of the block maxima approach, and also provides a sufficiently large data set for the BCGEV and POT approaches.

3.1.2 *Overview of Analysis*

The load effect data for three load effects, for bridge lengths of 20, 30, 40, and 50 m, and for each event type resulted in 41 sets of daily maxima. Load effects were noted for the different loading event types to allow application of the CDS method.

We consider there to be 250 working days per year, and extrapolate load effects to determine the return level for a return period of 1000 years, as specified by the Eurocode (EC1.2 2003). We stress that this is not the design life of the structure, which may be taken as 50 or 100 years. In such cases the probability load effect exceeding the return level is approximately 5% and 10% respectively.

3.1.3 *Box-Cox-GEV Model Analysis*

The BCGEV model is applied to the complete bridge length and load effect data set for a set of 11 thresholds. These thresholds are taken in steps of 0.5 standard deviations from $k = -2.5$ standard deviations to $k = +2.5$ standard deviations about the sample mean.

As Bali (2003) discusses, estimation of the model parameter, λ, is not generally robust. Therefore, to properly determine the behaviour of the model with respect to λ, the estimation was carried out for values of λ from 0 to 1 in steps of 0.01. The value of λ = 0 (the GPD) was approximated by setting λ = 0.001 for ease of numerical implementation.

In total, over 45 000 nonlinear regressions were performed to calculate the optimum fits for the BCGEV in this study.

3.2 *Application of the Box-Cox-GEV Model*

3.2.1 *Residuals and Model Parameters*

For each threshold and for each value of the model parameter, the SSR is calculated. Figure 1 shows this data for the first threshold, $k = -2.5$ (which is essentially all the data). Also shown in this figure is the mean SSR from the 41 sets of SSR for each λ. Figure 2 shows this mean SSR for each threshold and λ considered. As may be seen, the best fit to the data, on average, is for a threshold of $k = -1.5$ standard deviations from the sample mean and a model parameter of 0.98.

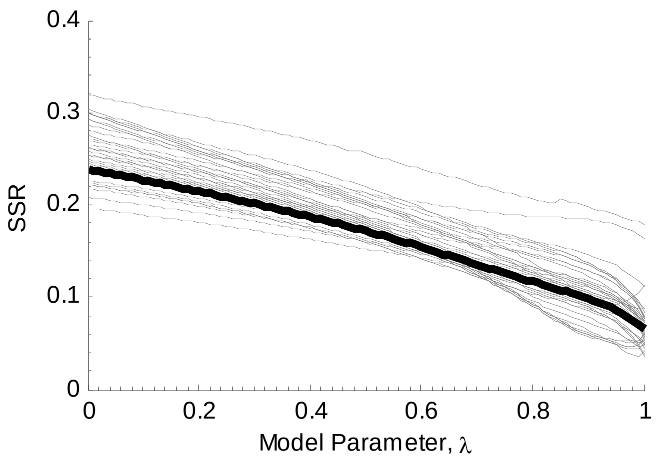

Figure 1. SSRs (gray lines) and mean SSR (heavy black line) for the lowest threshold, k = -2.5.

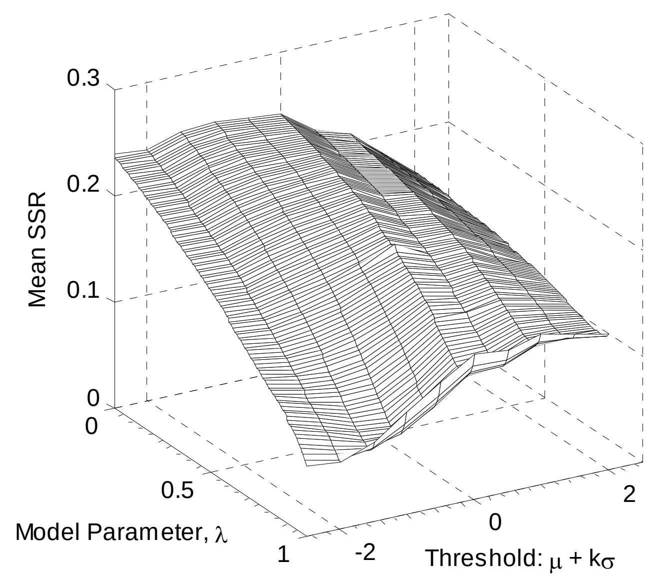

Figure 2. Mean SSR by threshold and model parameter.

Of particular interest are the values of the model parameter, λ, that minimize the SSR and provide the best fit to the data as these indicate the attraction of the data to the GEV or GPD domains. For each threshold, and for the 41 sets of data, the optimum model parameters are shown in Figure 3, along with their mean value of λ as it varies with threshold. Of note from this figure is that all values of λ are contained in the interval [0.91, 1]. Empirically then, it seems that the bridge traffic load effect considered is in the domain of attraction of the GEV, or block maxima, model. However, we must test the statistical significance of this finding.

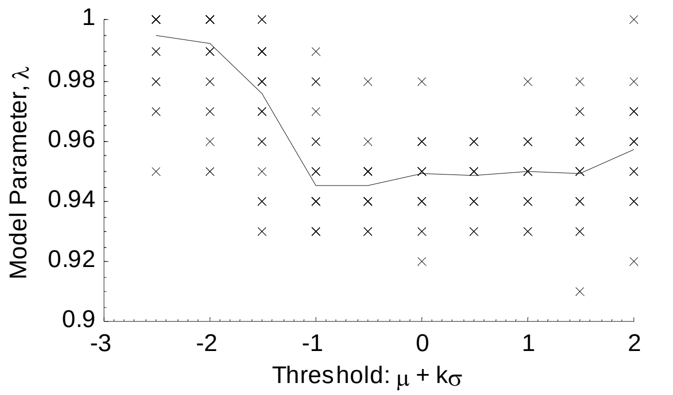

Figure 3. Mean SSR by threshold and model parameter (points); average of mean SSRs (line).

3.2.2 *Domain of Attraction Significance Testing*

Using the likelihood ratio test, the statistical significance of the BCGEV results is assessed. For each threshold, the 41 results are shown for the GEV and GPD models in Figures 4 and 5 respectively. Noting that the *y*-scale in these figures is logarithmic, these figures demonstrate that neither the GEV nor GPD models are appropriate within typical significance values for a wide range of thresholds. In particular, it is also evident that the GEV may be used within statistical significance for thresholds above $k \approx +1.5$; this is not true of the GPD model.

3.2.3 *Load Effect Prediction*

The optimum BCGEV parameter sets for each of the 41 data sets are used with the CDS method to predict lifetime load effect for the lengths and effects considered. With 250 days per year, the 1000-year return period corresponds to a probability of:

$$p^* = 1 - 1/(250 \cdot 1000) = 0.999996 \quad (9)$$

The return level is then found from:

$$s^* = H_C^{-1}\left(p^*\right) \quad (10)$$

where the CDS distribution is:

$$H_C(s) = \prod_{i=1}^{N} H_i(s) \quad (11)$$

and $H_i(\cdot)$ is the BCGEV distribution for loading event type *i*, given by Equation 4. A numerical root-finding algorithm is required to determine the return level due to the complexity of Equation 10.

The BCGEV lifetime load effect predictions are given in Figure 6 for the range of thresholds considered. It can be seen that the predictions are reasonably stable for thresholds in the range k = [-2.5,-1]. Once the threshold increases beyond the sample mean (k = 0) the predictions become unstable due to the reducing size of the data set.

Based on this last result, and the result that the model parameter is, on average, a minimum at a threshold of k = -1.5, it seems that a threshold of k = -1.5 is appropriate for application to daily maximum data. Therefore this threshold is used further in this study as a 'global optimum' threshold.

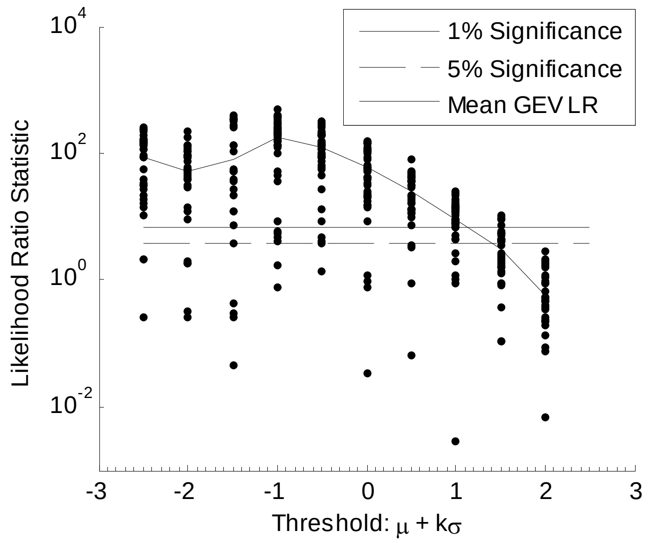

Figure 4. LR statistics for the GEV model, showing the mean LR value.

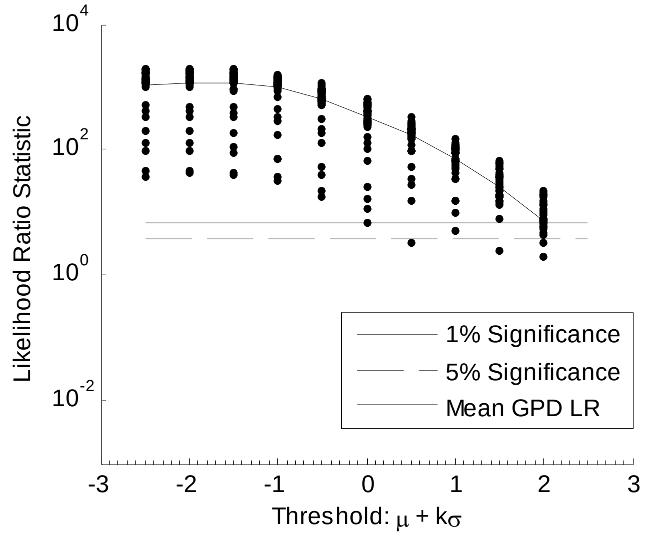

Figure 5. LR statistics for the GPD model, showing the mean LR value.

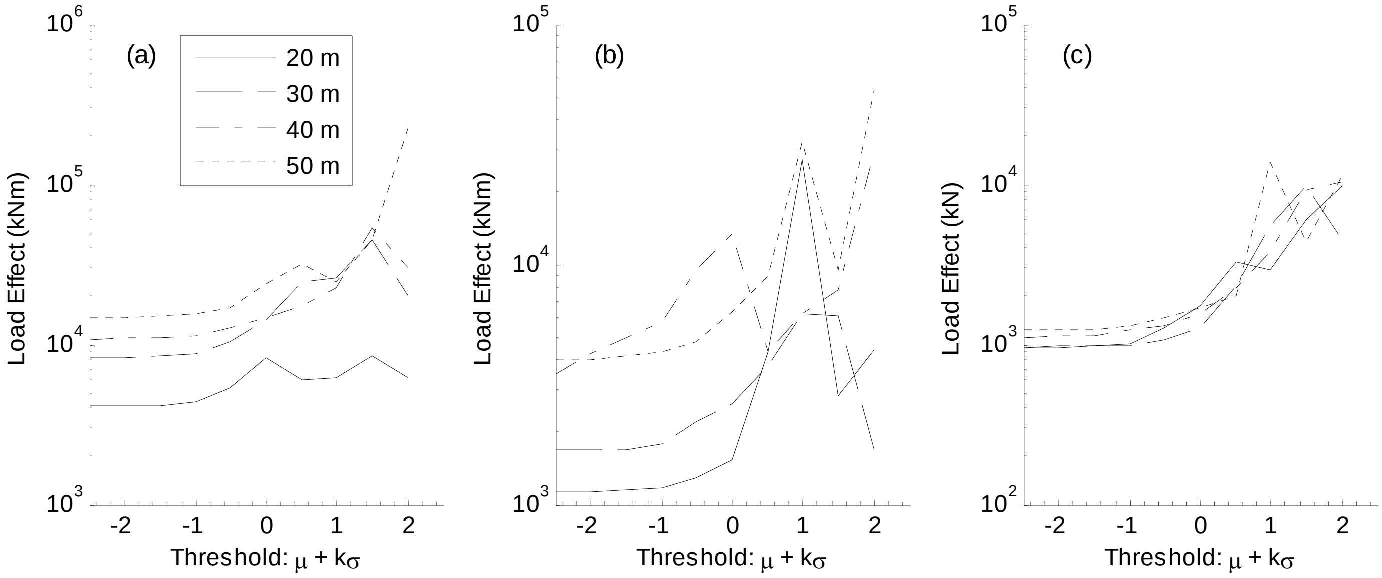

Figure 6. Box-Cox-GEV prediction of lifetime load effect for various thresholds, for Load Effects (a) 1, (b) 2, (c) 3.

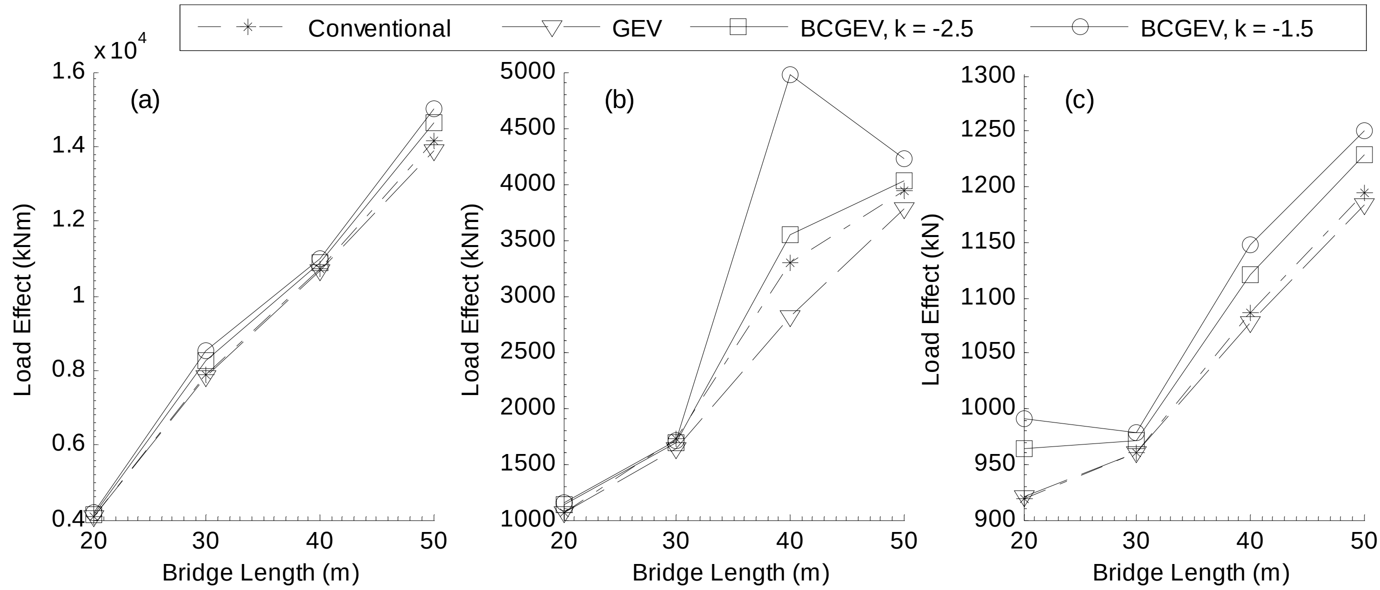

Figure 7. Comparison of load effect estimation methods for each load by bridge length, for Load Effects (a) 1, (b) 2, (c) 3.

3.3 *Comparison of Estimations*

3.3.1 *Basis*

Due to the results presented in Section 3.2, the GPD model is not considered further. Three methods for the estimation of bridge traffic load effect are next applied and the results compared:

1. The conventional approach which reflects current literature, as explained in Section 2.2. This method does not allow for the different distributions of loading event by type.
2. The block maxima approach, using the CDS formulation of Equations 2 and 3 applied to the complete set of daily maximum load effect data for each loading event type.
3. The Box-Cox-GEV distribution, using a CDS formulation to account for the different loading event types. Two thresholds will be considered: the 'global optimum' identified as k = -1.5; and the 'all data' value of k = -2.5, for a fairer comparison with the other non-threshold models.

The predicted load effects are given in Table 1.

3.3.2 *Results by Bridge Length*

Figure 7 shows the predictions for the various methods against bridge length. There is reasonable consistency of the estimation methods, but for Load Effect 2 at a bridge length of 40 m. For this load effect (bending moment over the central support of a two-span continuous bridge), and at such lengths a critical combination of small headway, vehicle length and influence line shape can occur. In this situation, 3- and 4-truck events mainly govern (Caprani 2005). One such loading event is shown in Figure 8.

The CDS distribution, and the comprising BCGEV fits (k = -1.5) to individual loading event-types are shown in Figure 9 for Load Effect 2, bridge length 40 m. It can be seen that 4-truck events, while not critical during the period of simulation, govern the extreme load effect. Further, the BCGEV distribution of the 4-truck event daily maximum data exhibits a Fréchet tail, which is unbounded. This accounts for the large return level predicted. However, such a tail is unreasonable since bridge loading events surely have a physical bound and therefore a Weibull (bounded) tail. Imposition of an optimization restraint to ensure a Weibull tail is possible (and should be done by practitioners) but is not done in this investigatory work.

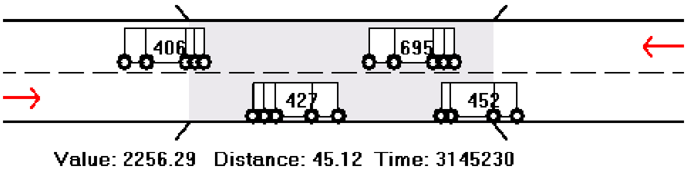

Figure 8. Sample 4-truck loading event on a 2-span 40 m long bridge – a critical combination for Load Effect 2 (GVW in kg/100 shown on trucks).

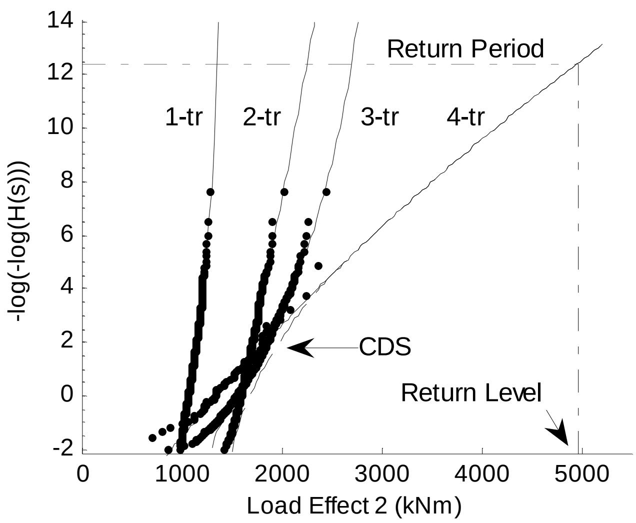

Figure 9. Load Effect 2, length 40 m Gumbel paper plot of individual loading event-type (1-truck, 2-truck etc.) data and BCGEV distributions and CDS distribution.

3.3.3 *Results by Percentage Comparison*

Using the conventional approach as the basis for comparing the impact of the models proposed in this paper, the percentage differences are given in Table 1 and plotted in Figure 10. From these sources there are two main points to note. Firstly, with only two exceptions, all predictions lie within +9% and -5% of the conventional value. Secondly, both BCGEV applications give higher load effects on average than the conventional method, in contrast to the GEV model which, on average, gives values lower than the conventional method.

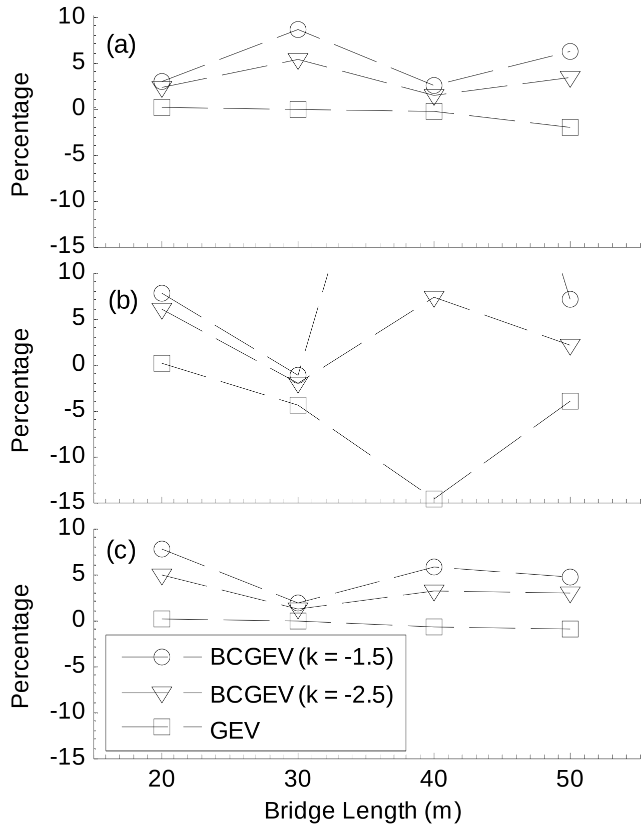

Figure 10. Differences in estimation methods for Load Effects (a) 1, (b) 2*, (c) 3. *+50.7% outlier for length 40 m not shown.

Table 1. Load effect predictions and percentage differences.

	Bridge Length (m)	Conventional Method	GEV with CDS*		Box-Cox-CGEV with CDS (*k* = -2.5)*		Box-Cox-CGEV with CDS (*k* = -1.5)*	
Load Effect 1 (kNm)	20	4059	4060	(0)	4152	(2.3)	4178	(2.9)
	30	7858	7848	(-0.1)	8275	(5.3)	8530	(8.6)
	40	10,733	10,691	(-0.4)	10,893	(1.5)	10,993	(2.4)
	50	14,141	13,860	(-2)	14,632	(3.5)	15,007	(6.1)
Load Effect 2 (kNm)	20	1064	1065	(0.1)	1129	(6.1)	1145	(7.6)
	30	1720	1643	(-4.5)	1685	(-2)	1699	(-1.2)
	40	3296	2812	(-14.7)	3536	(7.3)	4969	(50.7)
	50	3941	3782	(-4)	4026	(2.2)	4216	(7)
Load Effect 3 (kN)	20	920	920	(0.1)	965	(4.9)	991	(7.8)
	30	960	960	(0)	972	(1.2)	978	(1.8)
	40	1086	1077	(-0.8)	1121	(3.2)	1148	(5.7)
	50	1195	1185	(-0.9)	1229	(2.8)	1251	(4.7)
Mean Difference				(-2.3)		(3.2)		(4.9)**

* Percentage differences relative to the conventional method are given in parentheses.
** Excluding outlier of 50.7% for Load Effect 2, length 40 m - including this outlier gives a mean of 8.7% difference.

4 CONCLUSIONS

This paper has applied a recently-developed statistical model to the bridge traffic load effect problem. This Box-Cox-GEV model, through inference on the data itself, determines the most appropriate form of extreme value analysis. The generally subjective decision as to the choice of extreme value models, block maxima or peaks-over-threshold, is therefore avoided. In addition, a recently-introduced means of allowing for different loading event types – composite distribution statistics – has been applied to the Box-Cox-GEV model for a more sympathetic adherence to the underlying statistical mechanism. Comparison has been drawn between the load effect predictions that result from the application of a conventional approach, reflective of the literature, and the newly proposed modeling process.

In this study, it has been found that the Box-Cox-GEV model better fits the data than the competing GEV and GPD models with considerable statistical significance, for almost all thresholds considered. However, it has also been found that the data studied lies strongly in the domain of attraction of the GEV model. An optimum threshold level to apply to daily maximum load effect has been identified. Further, it was found that for thresholds near the sample mean and above, load effect predictions become unstable. For thresholds below this, predictions are quite stable and the model thus deemed robust in this region.

In comparison to the conventional and recently proposed (GEV) methods, application of the Box-Cox-GEV model gives higher load effects on average. Within the framework of composite distribution statistics, the Box-Cox-GEV model was found to be more sensitive to governing loading event types (as evidenced by the bridge length 40 m and Load Effect 2 prediction) than the competing GEV model.

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