Structural Analysis
Session 4

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Introduction

The aim of this research was to compare the calculated deflection between the joints of a continuous beam. The methods employed to find this were Macaulay brackets, virtual work and Mohr's moment-area theorem. These were all carried out subsequent to a stiffness analysis of a given beam.

The problems encountered which we had to over come were:

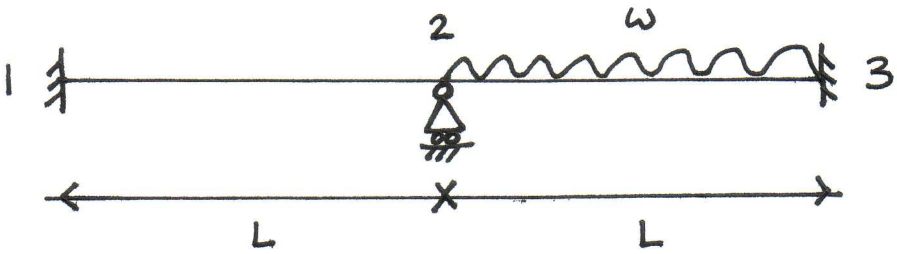
- No previous knowledge of the stiffness analysis approach.
- We had not analysed 4 degree indeterminate structures, to any great detail, beforehand.
- We were not sure how to find the deflection of such a beam using the methods listed above.

In order to better understand these concepts, we carried out a great deal of literature research and review.

Stiffness Analysis

Most structural analysis programs are based on the stiffness method. It is a repetitive, step-by-step method which is why computers handle it so well. However, for this very reason, when carrying out a stiffness analysis by hand, it can be quite cumbersome and the matrices become very large. Below is an example of a stiffness analysis for a continuous beam with 3 nodes. As can be seen, if the number of nodes was to increase or the support types were different, the matrix would become extremely big and difficult to handle, possibly leading to errors. The method used below is as outlined in Caprani (2008).

Example:



Firstly we construct the global matrix and restrict it.
We know that there can be no vertical displacement, horizontal displacement or rotation at nodes 1 or 3. Also, there can be no vertical or horizontal displacement at node 2. Therefore, the only degree of freedom is the rotation at node 2.

$$
\begin{array}{cc|ccc|ccc|ccc}
 & & \multicolumn{3}{c|}{1} & \multicolumn{3}{c|}{2} & \multicolumn{3}{c}{3} \\
 & & x & y & \theta & x & y & \theta & x & y & \theta \\
\hline
1 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
2 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
3 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\begin{pmatrix} \delta_{1x} \\ \delta_{1y} \\ \delta_{1\theta} \\ \delta_{2x} \\ \delta_{2y} \\ \delta_{2\theta} \\ \delta_{3x} \\ \delta_{3y} \\ \delta_{3\theta} \end{pmatrix}
=
\begin{pmatrix} W_{1x} \\ W_{1y} \\ W_{1\theta} \\ W_{2x} \\ W_{2y} \\ W_{2\theta} \\ W_{3x} \\ W_{3y} \\ W_{3\theta} \end{pmatrix}
-
\begin{pmatrix} \Sigma R_{1x} \\ \Sigma R_{1y} \\ \Sigma R_{1\theta} \\ \Sigma R_{2x} \\ \Sigma R_{2y} \\ \Sigma R_{2\theta} \\ \Sigma R_{3x} \\ \Sigma R_{3y} \\ \Sigma R_{3\theta} \end{pmatrix}
$$

From the above matrix configuration it can be seen that the system restricts as follows:

$$[a_{22}]\{\theta_2\} = \{w_{2\theta}\} - \{\sum R_{2\theta}\}$$

Equation 1

Member Contributions:

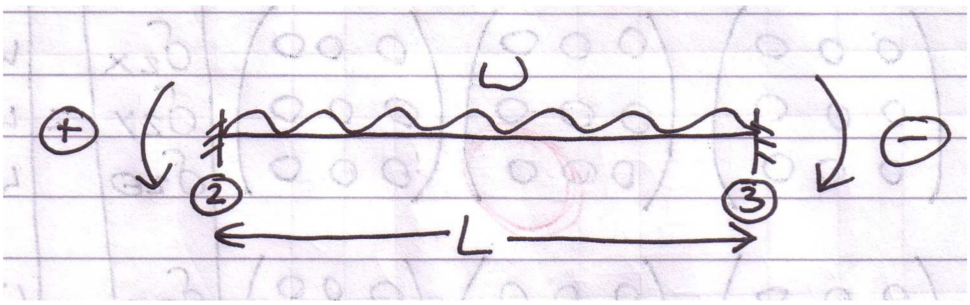
a_{22} = (term 3-3 of $K22_{12}$) + (term 3-3 of $K11_{23}$)

Where the K values are obtained from the member stiffness matrix.

$$\therefore a_{22} = \left(\frac{4EI}{L}\right)_{12} + \left(\frac{4EI}{L}\right)_{23} = \frac{8EI}{L}$$

Fixed-end Reactions:

These only apply to member 23 because it is the only loaded member.



$$M_{2_{23}} = \frac{wL^2}{12}$$

$$M_{3_{23}} = -\frac{wL^2}{12}$$

$$\therefore \sum R_{2\theta} = \frac{wL^2}{12}$$

Where $\sum R_{2\theta}$ = moments that are applied to node 2 by the member loads

There are no externally applied moments therefore, $w_{2\theta} = 0$.

Solve Equation:

Subbing the values into *Equation 1* we get:

$$\left[\frac{8EI}{L}\right]\{\theta_2\} = \{0\} - \left\{\frac{wL^2}{12}\right\}$$

$$\Rightarrow \theta_2 = -\frac{wL^3}{96EI}$$

Member End Forces:

Using the following formula we will find the member end forces:

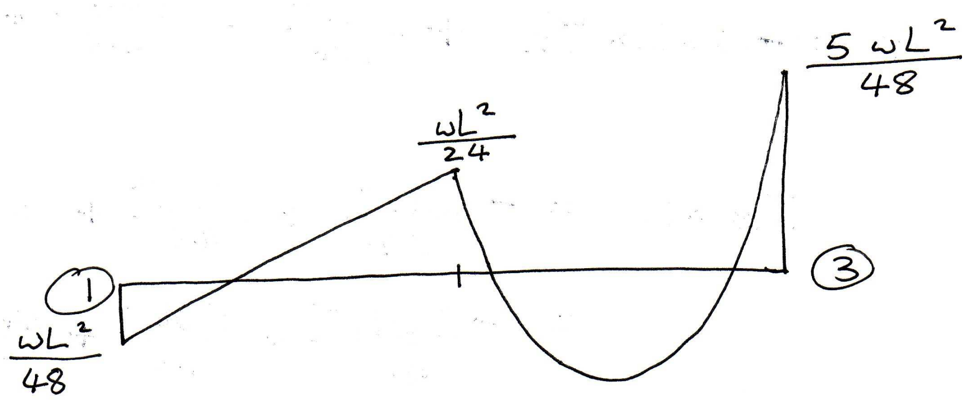
$$P_{1_{12}} = K11_{12}\delta_1 + K12_{12}\delta_2 + R_{1_{12}}$$

$$M_1 = 0 + \frac{2EI}{L} \times \frac{-wL^3}{96EI} + 0 = -\frac{wL^2}{48}$$

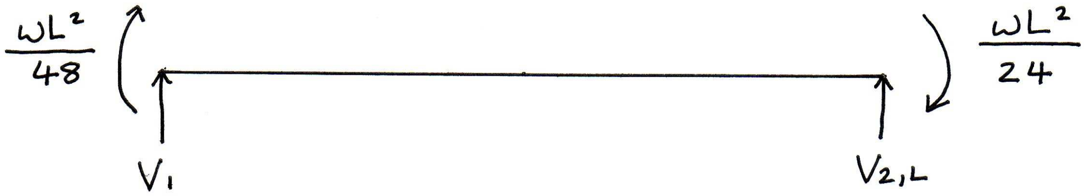
$$M_2 = \frac{4EI}{L} \times \frac{-wL^3}{96EI} = -\frac{wL^2}{24}$$

$$M_3 = \left(\frac{2EI}{L} \times \frac{-wL^3}{96EI}\right) - \frac{wL^2}{12} = -\frac{5wL^2}{48}$$

With this information we can determine the shear forces and draw the BMD.



Shear Forces on Member 12:



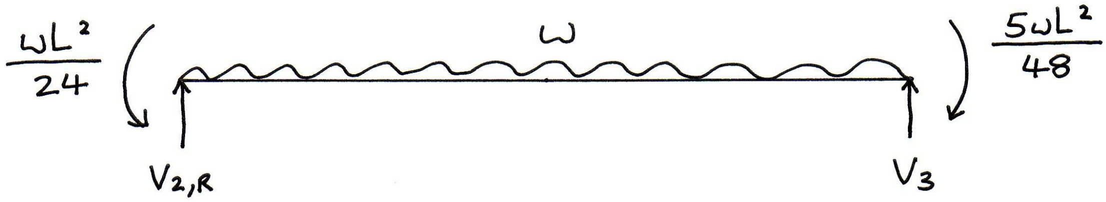
$$\sum M_{@1} = \frac{-wL^2}{48} - \frac{wL^2}{24} + (V_{2,L})(L) = 0$$

$$\therefore V_{2,L} = \frac{wL}{16}$$

$$\uparrow:\ V_1 + V_{2,L} = 0$$

$$\therefore V_1 = \frac{-wL}{16}$$

Shear Forces on Member 23:



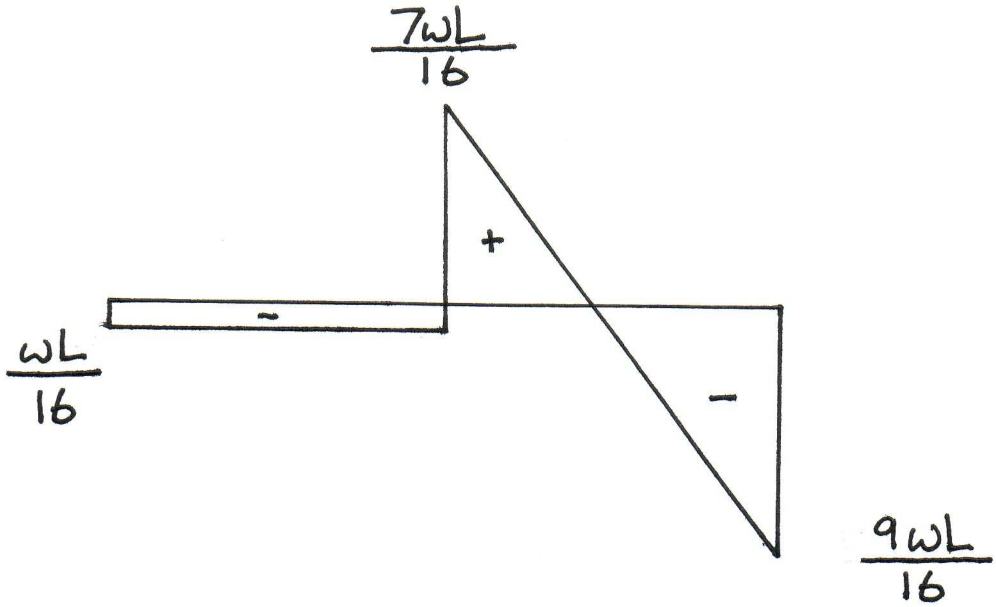
$$\sum M_{@2} = \left(-w \times L \times L/2\right) + \frac{wL^2}{24} - \frac{5wL^2}{48} + (V_3 \times L) = 0$$

$$\therefore V_3 = \frac{9wL}{16}$$

$$\uparrow:\ V_{2,R} - (w \times L) + \frac{9wL}{16} = 0$$

$$\therefore V_{2,R} = \frac{7wL}{16}$$

Using this information we can draw the SFD:

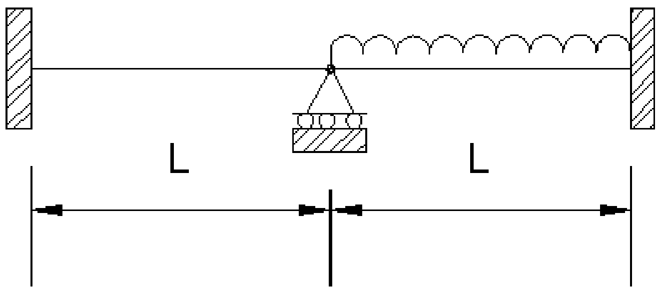


The beam is now fully analysed and the results of this stiffness analysis will be used to carry out the remainder of the project.

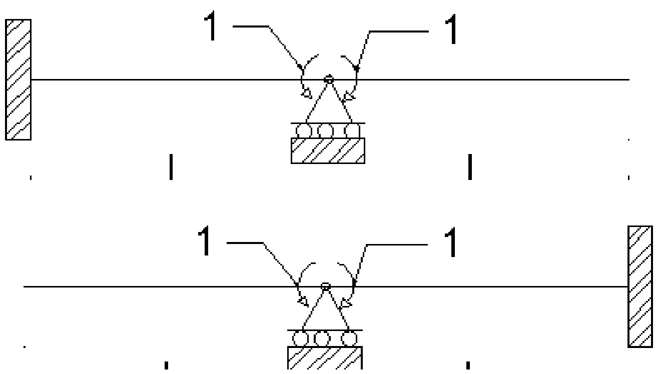
In order to try and confirm these results, a virtual work analysis of the beam was undertaken. This is out lined as follows in accordance with Caprani (2008).

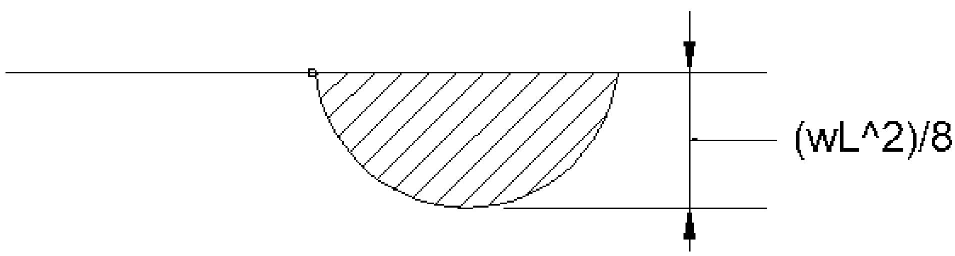
Example:

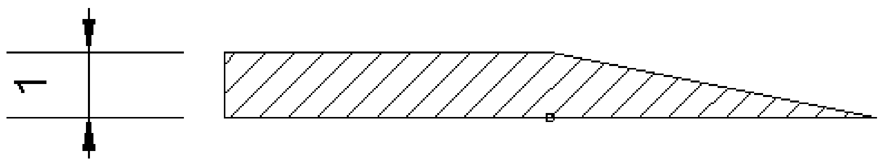
Because the structure is more than one degree indeterminate it must be broken up into two structures in order for to it to be analysed.



To analyse the beam it is assumed that there is a hinge at the pin support, a 1kNm moment is then placed at either side of this.







$$\int \frac{M^0 \times \delta M^1}{EI} = \frac{1}{EI}[0]_{ab} + \frac{1}{EI}\left[\frac{1}{3}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^0 \times \delta M^1}{EI} = \frac{-wL^3}{24EI}$$

$$\int \frac{M^0 \times \delta M^2}{EI} = \frac{1}{EI}[0]_{ab} + \frac{1}{EI}\left[\frac{2}{3}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^0 \times \delta M^2}{EI} = \frac{-2wL^3}{24EI}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{1}{EI}[(j)(k)(l)]_{ab} + \frac{1}{EI}\left[\frac{1}{3}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{4L}{3EI}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{1}{EI}[(j)(k)(l)]_{ab} + \frac{1}{EI}\left[\frac{1}{3}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{4L}{3EI}$$

$$\int \frac{M^1 \times \delta M^2}{EI} = \frac{1}{EI}\left[\frac{1}{2}(j)(k)(l)\right]_{ab} + \frac{1}{EI}\left[\frac{1}{2}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{L}{EI}$$

$$\int \frac{M^2 \times \delta M^1}{EI} = \frac{1}{EI}\left[\frac{1}{2}(j)(k)(l)\right]_{ab} + \frac{1}{EI}\left[\frac{1}{2}(j)(k)(l)\right]_{bc}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{L}{EI}$$

$$\int \frac{M^2 \times \delta M^2}{EI} = \frac{1}{EI}\left[\frac{1}{3}(j)(k)(l)\right]_{ab} + \frac{1}{EI}[(j)(k)(l)]_{bc}$$

$$\int \frac{M^1 \times \delta M^1}{EI} = \frac{4L}{3EI}$$

$$\frac{1}{EI}\begin{Bmatrix}\frac{-wL^3}{24}\\ \frac{-2wL^3}{24}\end{Bmatrix}+\frac{1}{EI}\begin{bmatrix}\frac{4L}{3} & L\\ L & \frac{4L}{3}\end{bmatrix}\times\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{Bmatrix}\frac{-wL^3}{24}\\ \frac{-2wL^3}{24}\end{Bmatrix}+\begin{bmatrix}\frac{4L}{3} & L\\ L & \frac{4L}{3}\end{bmatrix}\times\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{bmatrix}\frac{4L}{3} & L\\ L & \frac{4L}{3}\end{bmatrix}\times\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\begin{Bmatrix}\frac{-wL^3}{24}\\ \frac{-2wL^3}{24}\end{Bmatrix}$$

$$\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\frac{1}{\left(\left(\frac{4L}{3}\times\frac{4L}{3}\right)-(L\times L)\right)}\times\begin{bmatrix}\frac{4L}{3} & L\\ L & \frac{4L}{3}\end{bmatrix}\times\begin{Bmatrix}\frac{-wL^3}{24}\\ \frac{-2wL^3}{24}\end{Bmatrix}$$

$$\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\frac{9}{7L^2}\times\begin{bmatrix}\frac{2wL^4}{72}\\ \frac{-5wL^4}{72}\end{bmatrix}$$

$$\begin{Bmatrix}\propto_1\\ \propto_2\end{Bmatrix}=\begin{bmatrix}\frac{18wL^4}{504}\\ \frac{-45wL^4}{504}\end{bmatrix}$$

$$M = M^0+\propto.\delta M^1+\propto.\delta M^2$$

$$M_a = 0+\left(\left(\frac{18wL^4}{504}\right)\times(1)\right)+\left(\left(\frac{-45wL^4}{504}\right)\times(0)\right)$$

$$M_a=\frac{2wL^2}{56}$$

$$M_b = 0 + \left(\left(\frac{18wL^4}{504}\right) \times (1)\right) + \left(\left(\frac{-45wL^4}{504}\right) \times (1)\right)$$

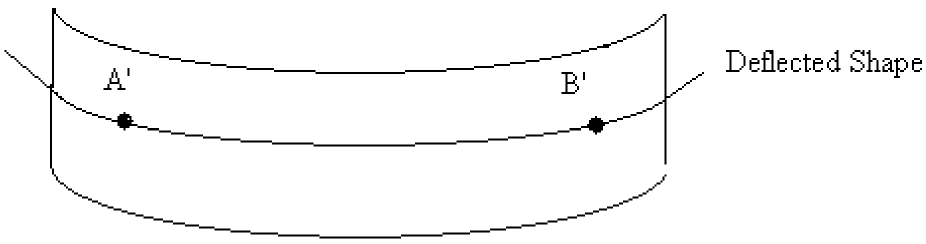
$$M_b = \frac{-3wL^2}{56}$$

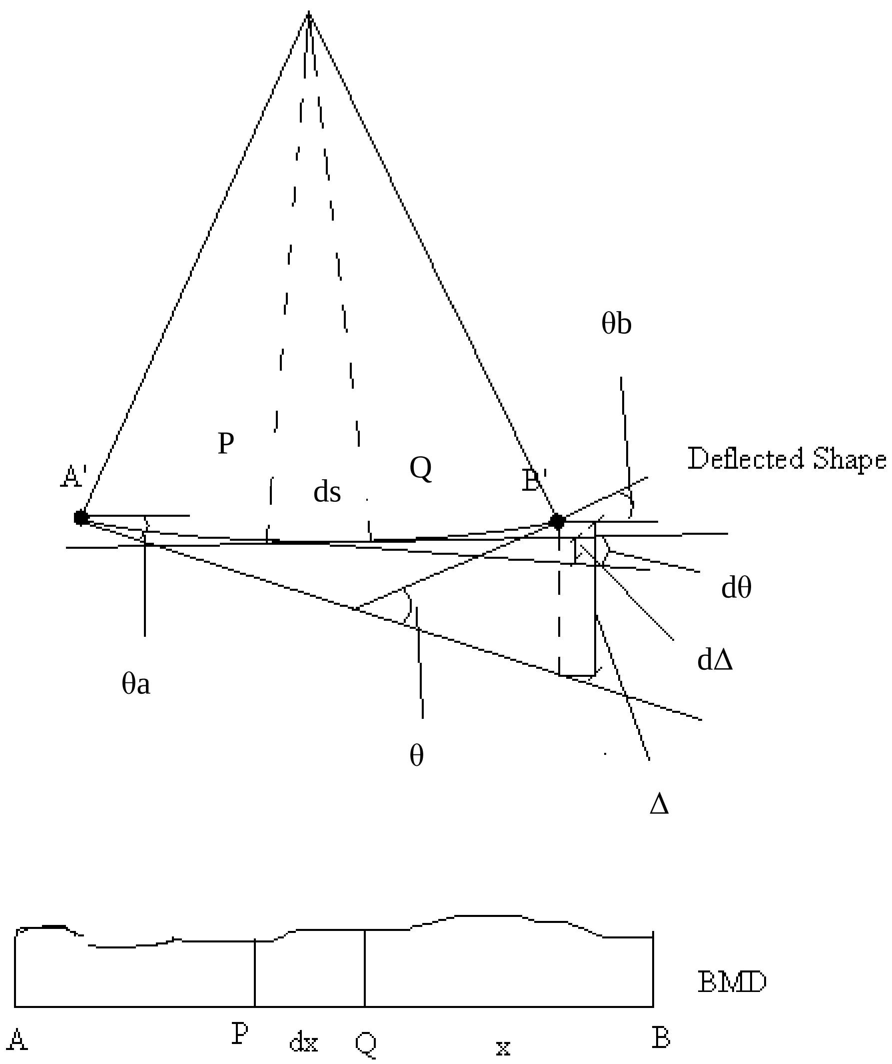
$$M_c = 0 + \left(\left(\frac{18wL^4}{504}\right) \times (0)\right) + \left(\left(\frac{-45wL^4}{504}\right) \times (1)\right)$$

$$M_c = \frac{-5wL^2}{56}$$

Moment-Area Method

The moment area method, which was developed by Mohr, is a powerful tool for finding the deflection of structures primarily subjected to bending. Its ease of finding deflections of determinate structures makes it ideal for solving indeterminate structures by using a method of compatibility of displacement. The method used is as outlined in Caprani (2007).





Ref: Caprani (2007)

Theory:

Consider a length of beam AB in its undeformed and deformed shape as shown above.

Note:

The original length of the beam = AB

The deformed length of the beam when it is loaded = A\`B\`

The angle at the centre of A\`OB\` is θ and this is the change in curvature from A\` to B\`

Dθ is the change in curvature from P to Q

M is the average bending moment over the portion dx between P and Q

The distance Δ is called the vertical intercept. It is the distance from B` to the tangent of A`. It is a vertical line measured perpendicular to the undeformed neutral axis.

Mohr's first Theorem:

Note: Angles are measured in radians

ds = R.dθ

Therefore: R = ds/dθ

Euler Bernoulli theory of bending gives: 1/R = M/EI

Therefore: dθ = (M/EI).ds

But: ds ≈ dx

for small deflection

Therefore:dθ = (M/EI).dx

Therefore: The total change in rotation between A and B is

$$\int_A^B d\theta = \int_A^B \frac{M}{EI} . dx$$

Where M/EI is the curvature

The diagram of this term as it changes along a beam is the curvature diagram (M/EI diagram)

Therefore: $d\theta = \theta_B - \theta_A = \int_A^B \frac{M}{EI} . dx$

In words this is: $[\text{Change in slope}]_{AB} = [\text{Area of M/EI diagram}]_{AB}$

Mohr's first theorem states: The change in slope over any length of a member subjected to bending is equal to the area of the curvature diagram over that length.

Mohr's second Theorem:

The diagram shows that $d\Delta = x.d\theta$

But: $d\theta = \frac{M.}{EI} dx$

Thus: $d\Delta = \frac{M.}{EI} .x.dx$

So for AB $\int_A^B d\Delta = \int_A^B \frac{M}{EI} . x. dx$

Therefore: $\Delta_{BA} = [\int_A^B \frac{M}{EI} dx] \, x$

= First moment of M/EI diagram about B

In words this is:

$[\text{Vertical Intercept}]_{BA} = [\text{Area of M/EI diagram}]_{BA} \, x$

[Distance from support B to centroid of $(M/EI)_{BA}$ diagram]

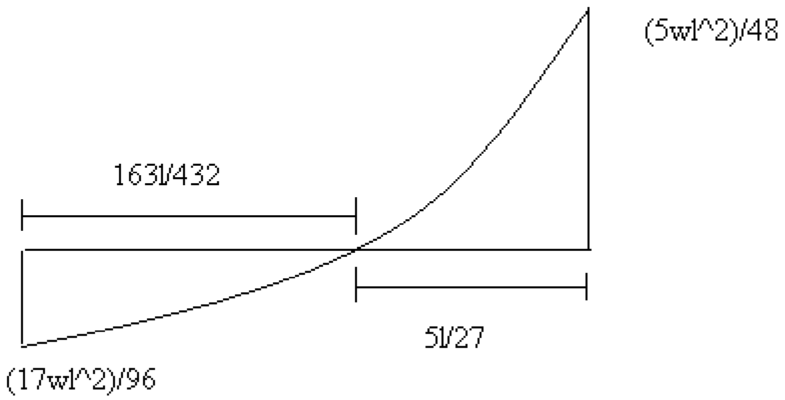
Mohr's second Theorem states that for an originally straight beam, subject to bending moment, the vertical intercept between one terminal and the tangent to the curve of another terminal is the first moment of the curvature diagram about the terminal where the intercept is measured.

Note: The vertical intercept is not the deflection.

Calculations:

The following is the BMD for the beam with the UDL to the right of the Max moment.

I will now use this and Mohr's second theorem to calculate the deflection under the $17wl^2/96$ bending moment



Note: $\Delta = \delta$ as θ_c is zero

$EI\ \Delta_{max} = [(1/3*5l/27*5wl^2/48)*(163l/432 + (3/4*5l/27))] –$

$[(2/3*163l/432*17wl^2/96)*(3/8*163l/432)]$

Therefore:

$$\Delta_{max} = \frac{-2.98e-3wl^4}{EI}$$

Theory of Macaulay’s Method

The Macaulay’s Method is a means of finding the equation that describes the deflected shape of a structure. This is done by combining the Macaulay’s theory of obtaining a single equation for the bending moment with the Euler-Bernoulli theory in which the bending moment equation is integrated to find the deflection equation. It is outlined below in accordance with Caprani (2007).

Background information

On investigating the Euler-Bernoulli theory of bending at a certain location along a beam we found that

$$\frac{I}{R} = \frac{M}{EI}$$

Where R is the radius of curvature of a point and mathematically we know that

$$\frac{I}{R} = \frac{d^2y}{dx^2}$$

Where y is the deflection at a point and x is the distance of this point along the beam. Thus combining this knowledge we can say that

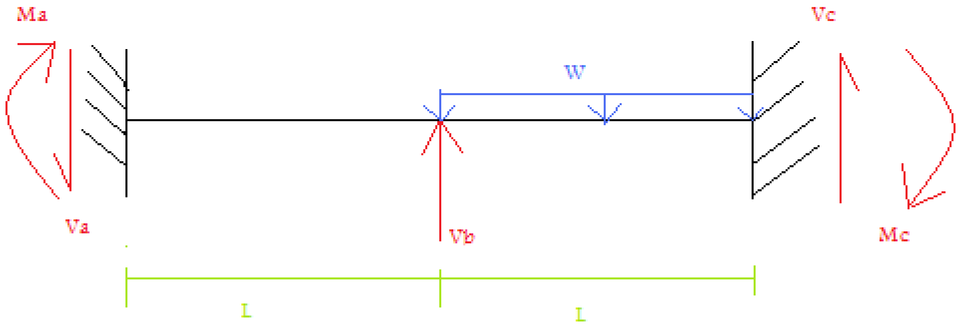
$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

And this is the fundamental equation in finding deflection.

Another key ingredient in the Macaulay’s method is the Discontinuity Function. It gives the equation the ability to switch on and off functions when they are needed so that the equation produces the correct answer throughout the beam. This is achieved by the special mathematical characteristics of the discontinuity function where its value is assumed to be zero when the result within the brackets is a negative value.

<u>Calculations</u>

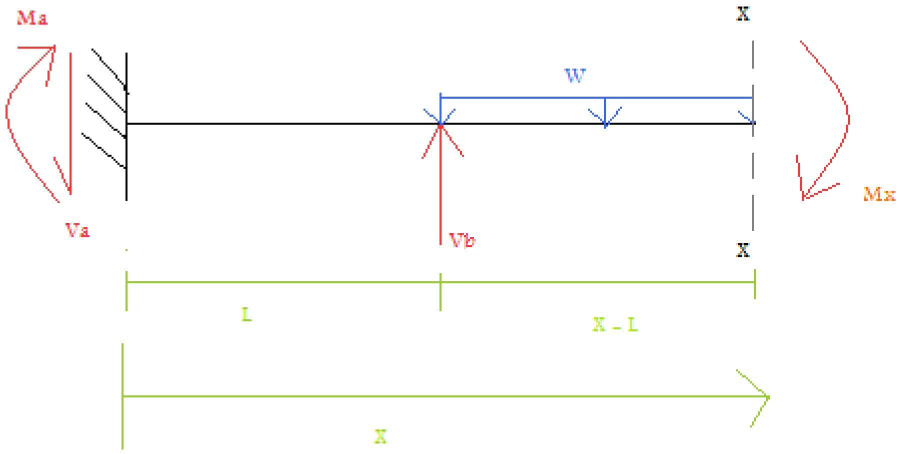
Reactions obtained from stiffness matrix



$$Ma = \frac{WL^2}{48} \qquad Mc = \frac{5WL^2}{48}$$

$$Va = \frac{WL}{16} \qquad Vb = \frac{W}{2} \qquad Vc = \frac{9WL}{16}$$

Cut section of Beam for Macaulay Calculation



Find Bending Moment at x using Macaulay's.

$$EIMx = EI\frac{d^2y}{dx^2} = \frac{WL^2}{48}[x]^0 - \frac{WL}{16}[x]^1 + \frac{WL}{2}[x-L]^1 - \frac{w}{2}[x-L]^2$$

Integrate this to find rotation.

$$EI\frac{dy}{dx} = \frac{WL^2}{48}[x]^1 - \frac{WL}{32}[x]^2 + \frac{WL}{4}[x-L]^2 - \frac{w}{6}[x-L]^3 + C_1$$

Integrate again to find Deflection.

$$EIy = \frac{WL^2}{96}[x]^2 - \frac{WL}{96}[x]^3 + \frac{WL}{12}[x-L]^3 - \frac{w}{24}[x-L]^4 + C_1 + C_2$$

The constants of integration can be solved from our boundary conditions.

$\frac{dy}{dx} = 0$ at $x = 0$

$\therefore C_1 = 0$

And y = 0 at x = 0

$\therefore C_2 = C_1 = 0$

Thus are equations for rotation and deflection are;

$$EI\frac{dy}{dx} = \frac{WL^2}{48}[x]^1 - \frac{WL}{32}[x]^2 + \frac{WL}{4}[x-L]^2 - \frac{w}{6}[x-L]^3$$

And

$$EIy = \frac{WL^2}{96}[x]^2 - \frac{WL}{96}[x]^3 + \frac{WL}{12}[x-L]^3 - \frac{w}{24}[x-L]^4$$

To find deflection at max moment sub in distance from left support to max moment which is $x = \frac{23L}{16}$

$$EIy = \frac{WL^2}{96}\left[\frac{23L}{16}\right]^2 - \frac{WL}{96}\left[\frac{23L}{16}\right]^3 + \frac{WL}{12}\left[\frac{23L}{16} - L\right]^3 - \frac{w}{24}\left[\frac{23L}{16} - L\right]^4$$

$$\therefore y = \frac{-57.1x10^{-4}WL^4}{EI}$$

Virtual Work

Background

The following was carried using Caprani (2008) and Davies (1982)

The law of the conservation of energy stated “energy can neither be created nor destroyed only changed from one from to another” for structures this can be stated that “a structural system that is isolated such that neither gives nor receives energy the total energy of this system remains constant”.
As stated above the structure in question must be isolated, this means all sources of restraint and loading must be identified and accounted for. For instance to neglect a structures self weight could be hazardous as the structure would receive a gravitational energy and not accounting for this could induce a structural collapse.
The virtual work method of structural analysis is based around the idea that Strain energy is equivalent to the amount of deformation causes by an external load, which means strain energy is the amount of energy stored in a member due to the deformation created by an external load. Therefore a small increase in the force applied to structure results in a small deflection. The work done can be seen to be (force X displacement) the average force during the structures course of deflection times, the displacement causes during this time.
Virtual work is based upon the principle of minimum total potential energy, it can be seen that any small variation about the equilibrium must do no work. Thus the Principle of Virtual Work States that
“A structure is in equilibrium if and only if, the virtual work of all the forces acting on the structure is zero”.
The term virtual in when referring to “Virtual Work” means having the effect of but the effect of but not the actual form of what is specified, which means that ways impose virtual work can be imposed without having to worry about how this would be achieved in the physical world, this makes it a very power full tool for the engineers arsenal.

Calculating Max Deflection:
To calculate the max deflection of the beam a Compatibility and Equilibrium set must be established.

- A Compatibility set: is formed from the actual deflections an rotations that occur along the length of the beam.

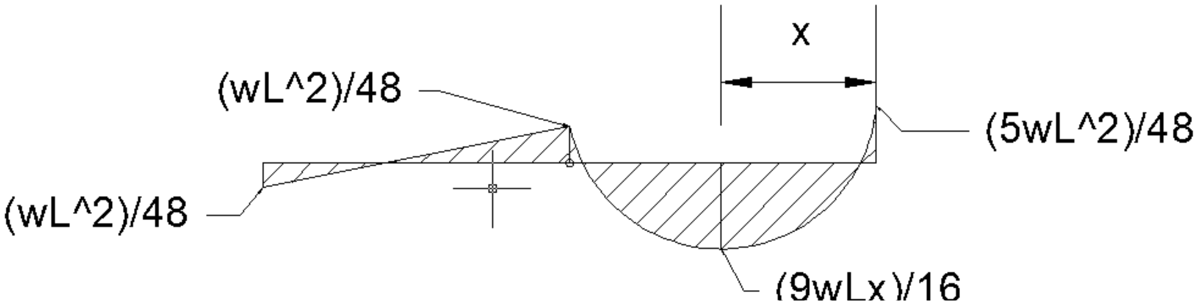
- An Equilibrium Set: is based on a unit virtual force is applied, which is in equilibrium with the internal virtual moment it causes.

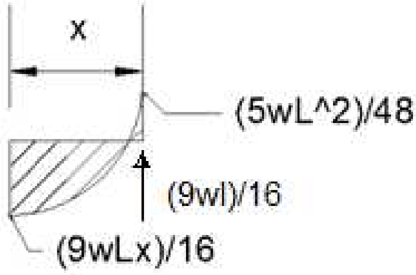
Compatibility set.

To calculate the rotations along the length of the beam, the external deflection at distance X must be found.

$$\theta = \int_0^L \frac{M_x}{EI_x} dx$$

From the stiffness analysis it can be calculated.





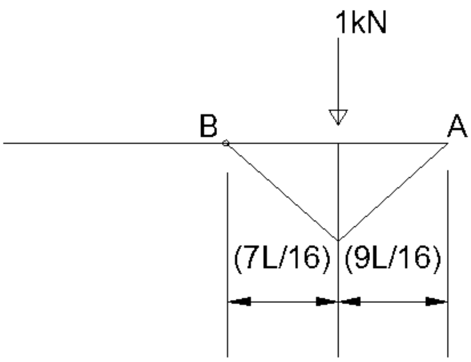
$\sum$ of the moments about x = 0

=> $$M(x) - \left(\frac{9wL}{16}\right)(x) = 0$$

=> $$M(x) = \left(\frac{9wLx}{16}\right)$$

Equilibrium Set:

As the value for $\delta F = 1$ was chosen, only the virtual moments are left to be calculated.



$\sum$ of the moments about A = 0

=> $$\frac{-9}{16}(1) + V_b(L) = 0$$

=> $$V_b = \left(\frac{9}{16L}\right)$$

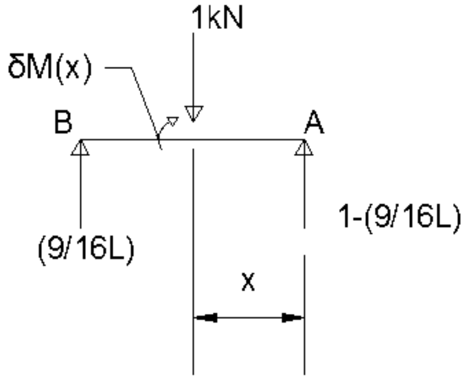
$\sum$ fx = 0

=> $$V_a + V_b - 1 = 0$$

=> $$V_a + \frac{9}{16L} - 1 = 0$$

=> $V_a = \left(1 - \frac{9}{16L}\right)$

$\sum$ of the moments about x = 0 section (a-x)



=> $\delta Mx^1 = \left(1 - \frac{9}{16L}\right)(x)$

=> $\delta Mx^1 = \left(x - \frac{9x}{16L}\right)$

$\sum$ of the moments about x = 0 section (b-x)

=> $\delta M_x^1 = \left(\frac{9}{16L}\right)(L - x)$

=> $\delta M_x^2 = \left(\frac{9}{16} - \frac{9x}{16L}\right)$

Virtual Work Equation:

$\delta W = 0$

$\delta W_E = \delta W_I$

$\Sigma y_i.\delta F_i = \sum \theta_i \delta M_i$

Substitute in the values for the real rotations and the virtual moments.

$$y.1 = \int_0^{\frac{9L}{16}} \left(\left[\frac{M_x}{EI}\right].\delta M_x^1\right)dx + \int_0^{\frac{7L}{16}} \left(\left[\frac{M_x}{EI}\right].\delta M_x^2\right)dx$$

$$y = \left[\frac{9wLx^3}{48EI} - \frac{81wLx^3}{48EI}\right]_0^{\frac{9L}{16}} + \left[\frac{81wLx^2}{512EI} - \frac{81wLx^3}{768}\right]_0^{\frac{7L}{16}}$$

$$y = \left(\frac{9wL\left(\frac{9L}{16}\right)^3}{48EI} - \frac{81wLx\left(\frac{9L}{16}\right)^3}{48EI}\right) + \left(\frac{81wl\left(\frac{7L}{16}\right)^2}{512EI} - \frac{81wl\left(\frac{7L}{16}\right)^3}{768}\right)$$

$$y = \frac{-0.2757wL^4}{EI} + \frac{0.0303wL^3}{EI}$$

Conclusion

Carrying out this project has led to a good collective understanding of the stiffness analysis approach for continuous beams. This is very helpful as this topic was covered for the project prior to our starting it in class, therefore giving us a good basis for understanding the more complex structures which could be encountered in the course.
The results from the stiffness analysis were used to obtain the deflection, at the point of maximum moment, when utilizing Macaulay's, virtual work and the moment-area method.

The deflection obtained from the moment-area method came out as the following:

$$y = \frac{-2.98x10^{-3}WL^4}{EI}$$

Due to the assumptions made, in regard to the areas of the bending moment diagram, the above equation may not be correct.

The deflection obtained from the Macaulay method came out as the following:

$$y = \frac{-57.1x10^{-4}WL^4}{EI}$$

There are discrepancies in this answer due to generalization of certain terms. For example, the removal of the constants of integration due to the boundary conditions. Also, axial load was not taken into account.

The deflection obtained from the virtual work method came out as the following:

$$y = \frac{-0.2757wL^4}{EI} + \frac{0.0303wL^3}{EI}$$

The choice of compatibility set may have adversely affected this answer. Also, a hinge was assumed at the middle support in this compatibility set.

References

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Note: The books/notes listed above that are not directly referenced within the report were read to better our understanding of the problems at hand. No part of them were actually used in the report.

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