Investigation of strip footings under column loads

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Introduction (Essentials of a good Foundation)

The foundation is the supporting part of a structure. The term is usually restricted to the member that transmits the superstructure load to the earth, but in its complete sense it includes the soil and rock below. It is a transition or structural connection whose design depends on the characteristics of both the structure and the soil and rock. A satisfactory foundation must meet these requirements:

1. It must be placed at an adequate depth to prevent frost damage, heave, undermining by scour, or damage from future construction nearby.
2. It must be safe against breaking into the ground.
3. It must not settle enough to disfigure or damage the structure.

These requirements should be considered in the order named. The last two are capable of reasonably accurate determination through methods of soil and rock mechanics, but the first involves consideration of many possibilities, some far beyond the realm of engineering.

During the long period of time that the ground must support a structure, it may be changed by many man made and natural forces. These should be carefully evaluated in choosing the location for a structure and particularly in selecting the type of foundation and the minimum depth to which it must be extend.

Continuous pad and beam foundations

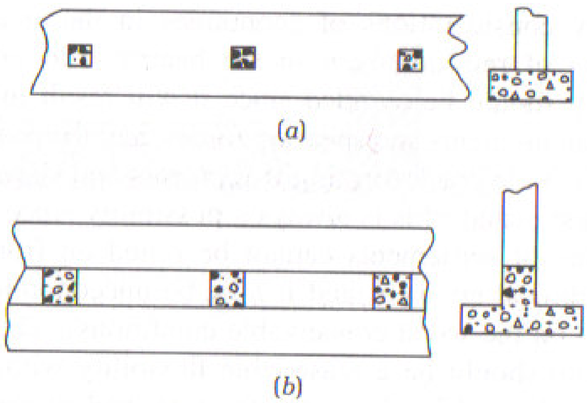
It may often be more economical to construct the foundations of a row of columns as a row of pad foundations with only a joint between each pad, rather than to provide individual excavations at a close spacing. Foundations of this type with individual but touching pads are more economical in reinforcing steel than continuous beam foundations, since the latter require a good deal of reinforcement to provide for the stresses due to differential settlement between adjacent columns.

However, continuous beam foundations may be required to bridge over weak pockets in the soil or to prevent excessive differential settlement between adjacent columns. The advantages of the continuous pad or beam foundation are:

1. Ease of excavation by back-acter or other machines.
2. Any formwork required can be fabricated and assembled in longer lengths.
3. There is improved continuity and ease of access for concreting the foundations.
4. These foundations provide strip foundations for panel walls of the ground floor of a multi story framed building.

Structural design of Continuous Beam Foundations

Continuous beam foundations may take the form of simple rectangular slab beams or for wider foundations with heavy loads, inverted T beams.



Continuous beam foundations. (a) Rectangular slab. (b) Inverted T beam

Structural design problems are complicated by factors such as varying column loads, varying live loads on columns, and variations in the compressibility of the soil. In most cases it is impossible to design the beams on a satisfactory theoretical basis. In practise, soil conditions are rarely sufficiently uniform to assume uniform settlement of the foundations, even though the column loads are equal.

Inevitably there will be a tendency to greater settlement under one individual column, which will then transfer a proportion of the load to the soil beneath adjacent columns until the whole foundation eventually reaches equilibrium. The amount of load transfer and of yielding of individual parts of the foundation beam is determined by the flexural rigidity of the beam and the compressibility of the soil considered as one unit.

For reasonably uniform soil conditions and where maximum settlement will, in any case, be of a small order, a reasonably safe design method is to allow the maximum combined dead and live load on all columns, to assume uniform pressure distribution on the soil, and design the foundation as an inverted beam on unyielding columns. However if the compressibility of the soil is variable, and if the live load distribution on the columns can vary, this procedure could lead to an unsafe design.

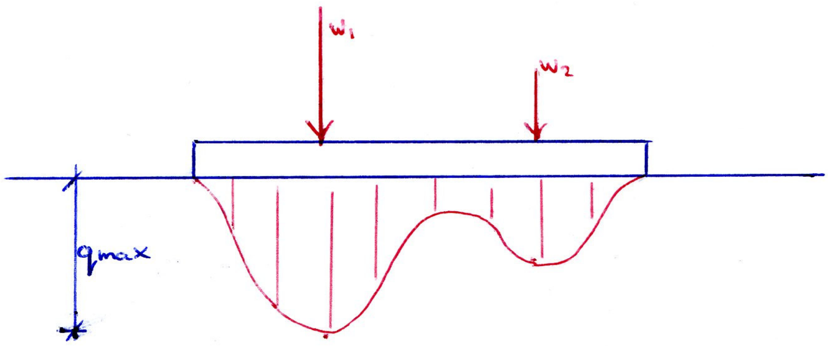
The structural engineer must then obtain from the geotechnical engineer estimates of maximum and differential settlements for the most severe conditions of load distribution in relation to soil characteristics. The geotechnical engineer must necessarily base his estimates on complete flexibility in the foundation, and the structural engineer then designs the foundation beam on the assumption of a beam on yielding supports.

The degree of rigidity which must be given to the foundation beam is governed by the limiting differential movements which can be tolerated by the superstructure and by economies in the size and amount of reinforcement in the beams. Too great a rigidity should be avoided since it will in high bending moments and shearing forces, and the possibility of a wide crack forming if moments and shears are underestimated (this is always a possibility since close estimate of settlements cannot be relied on from the geotechnical engineer and it may be uneconomical to design on the worst conceivable conditions). The general aim should be a reasonable flexibility within the limits tolerated by the superstructure, and in cases of high bending moments the junctions of beams, slabs, and columns should be provided with generous splays and haunches to avoid concentrations of stress at sharp angles.

Contact Pressure

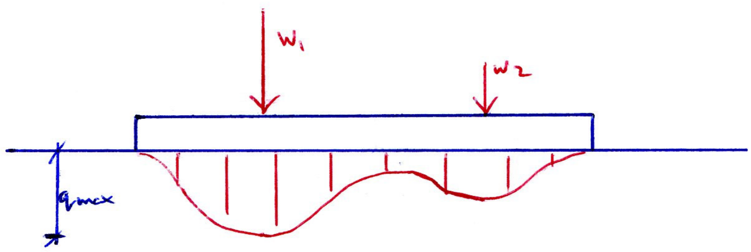
Contact pressure is the intensity of loading transmitted from the underside of the loading of a foundation to the soil. The distribution of contact pressure depends on both the rigidity of the footing and on the stiffness of the soil under the foundation.

1- Footing carrying concentrated column loads on **Rock:**



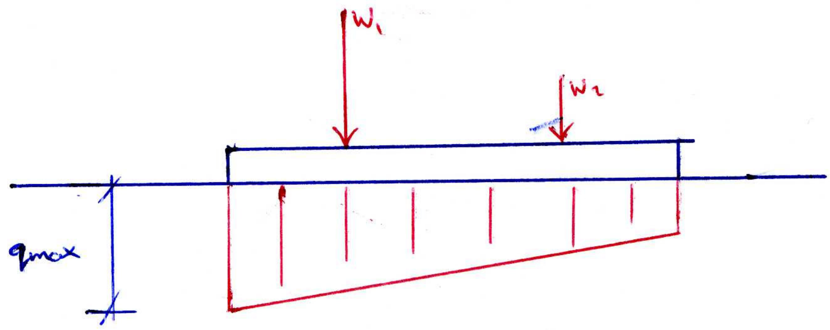
The rock has a high stiffness modulus, therefore the load is transferred to a relatively small area, since a high intensity of stress can develop.

2- **Stiff Soil:**



On a less stiff foundation, the loading is distributed laterally. This produces lower values of contact pressure, which results in the stresses underneath both column loads being a lot closer in magnitude.

3- **Soft Soil:**

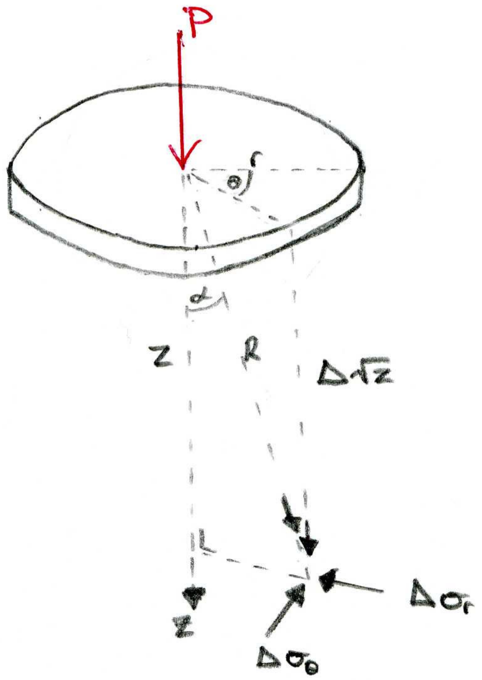


When the loading is on a footing in contact with a soft soil, the low value of the contact pressure results in an almost uniform distribution of the soil pressure.

Aside: For the purpose of calculating stresses and displacements within the soil mass, sufficient accuracy may be obtained by assuming a uniform distribution of contact pressure.

Stresses due to a vertical load (P)

With reference to the original works of Bousineq. Using polar coordinates (r, θ, z), the increases in stress at a given point due to the point load:



$$\Delta\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

$$\Delta\sigma_R = \frac{P}{2\pi R^2}\left(\frac{3zr^2}{R^3} - \frac{R(1-2\nu')}{R+z}\right)$$

$$\Delta\sigma_\theta = \frac{P}{2\pi R^2}(2\nu'-1)\left(\frac{z}{R} - \frac{R}{R+z}\right)$$

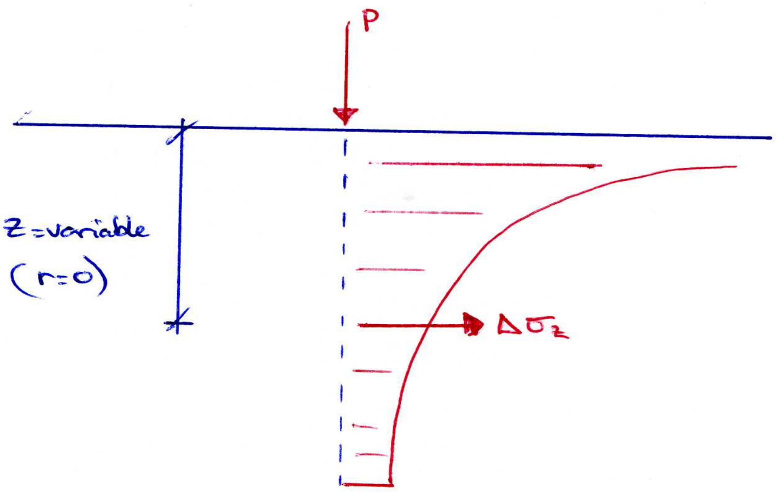
For undrained conditions (water drains away casusing an increase in pore pressure):

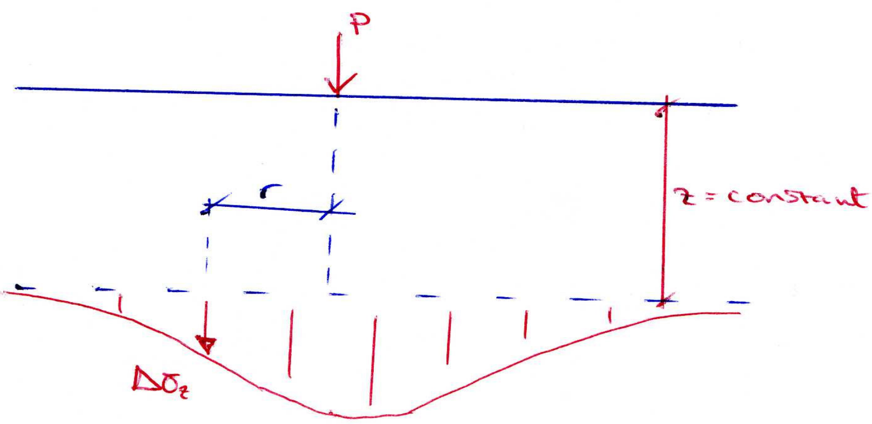
$$r = 0.5 \Rightarrow \Delta\sigma'_\theta = 0 \,, \quad \Delta\sigma_r = \Delta . \sigma_z . \frac{r^2}{z^2}$$

$$\Rightarrow \Delta\sigma_z = \frac{P}{z^2} . I_p$$

Where I_p is the point load influence factor:

$$I_p = \frac{3}{2\pi}\left(\frac{z}{R}\right)^5 = \frac{3}{2\pi}\left(\frac{1}{1+(r/z)^2}\right)^{5/2} \quad \ldots \quad R = (r+z)^{1/2}$$





Stress Distribution under vertical column load

When a load P acting on the surface of a footing of finite length, a vertical stress σ_z is obtained in addition to lateral and shear stress. The equation we have decided to use is Boussinesq's Equation. This states that the amount of stress varies due to the depth of the soil and the distance away form the reaction.

BOUSSINESQ'S FORMULA

$$\sigma_z = \frac{3P}{2\pi} \times \frac{z^3}{R^5} \quad \ldots\ldots \quad R = \sqrt{z^2 + r^2}$$

$$\sigma_z = \frac{3P}{2\pi} \times \frac{z^3 \cdot z^2}{R^5 \cdot z^2}$$

$$= \frac{3P}{2\pi z^2} \cdot \frac{z^5}{R^5}$$

$$= \frac{3P}{2\pi z^2} \times \frac{1}{\left(\frac{r^2 + z^2}{z^2}\right)^{5/2}} \quad \ldots\ldots \quad R^5 = (r^2 + z^2)^{5/2}$$

$$= \frac{3P}{2\pi z^2} \times \frac{1}{\left[\left(\frac{r^2}{z^2}\right) + \left(\frac{z^2}{z^2}\right)\right]^{5/2}}$$

$$= \frac{3P}{2\pi z^2} \times \left[\frac{1}{1+\left(\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$= \frac{P}{z^2} \cdot \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\Rightarrow \boxed{\sigma_z = \frac{P}{z^2} \times I_p}$$

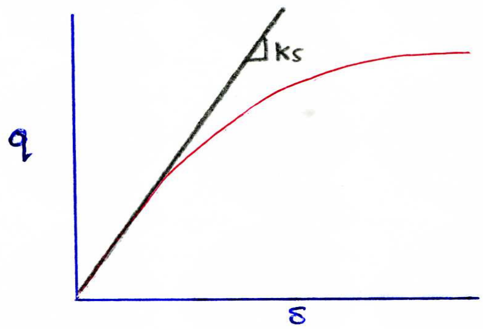
WHERE I_p = INFLUENCE FACTOR $= \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$

Modulus of Subgrade Reaction

The coefficient of subgrade reaction otherwise known as 'The Modulus of Subgrade Reaction', represented by **k_s**, is a mathematical constant that represents the foundation's stiffness (i.e. the overall stiffness of the foundation itself and the soil bearing it). Fundamentally it is the relationship of the soil pressure at a given point (q) to the settlement of the foundation at the same point (δ).

$$\mathbf{k_s = q/\delta}$$

There are several ways in which k_s can be calculated. It can be found experimentally, by loading a unit area of the foundation and plotting the load against the settlement, the following diagram illustrates this:



The value of k_s is also affected by footing size, shape and depth. Terzaghi proposed the following expressions:

For Cohesive Soils, $K_s = \dfrac{K_1}{B}$ For Granular Soils, $K_s = K_1\left[\dfrac{(B+1)}{(2B)}\right]^2$

Accounting for depth: $K_s = K_1\left[\dfrac{B+1}{2B}\right]^2\left[\dfrac{1+2D}{B}\right]$

For Rectangular footing, L×B on granular soil: $K_s = K_1\dfrac{(1+0.5B/L)}{1.5}$

where L, B, D = footing length, width, depth respectively

K_s = value for a full sized footing

K_1 = value obtained from unit area load tests.

The q vs δ curve above is very similar to a typical stress-strain diagram. Hence, Vesic proposed the following relationship which relates the stiffness of the concrete foundation to the Young's Modulus of the soil:

$$K_s = \left[0.65 E_s / (1-\mu^2)\right] \sqrt[12]{E_s B^4 / EI}$$

E_s = modulus (of elasticity) of the soil

μ = Poisson ratio of the soil

E = modulus of elasticity of the beam

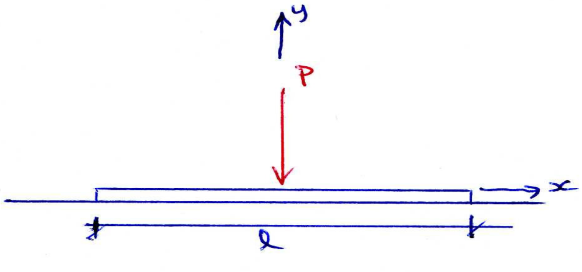
I = moment of inertia of beam's cross section

Winkler also produced an approach back in 1867 that is still used today. He proposed the use of a bed of springs to replace the elastic soil supporting the foundation. Therefore the vertical reaction at any point q is equal to $k_s y$, where y is the deflection. This is basically differential equation for the deflection curve of a beam, which can be used to relate the stiffness of the concrete foundation to the induced soil pressure:

$$EI \frac{d^4y}{dx^4} = q = -ky \qquad \text{where} \quad k = K_s \times B$$

B = width of footing

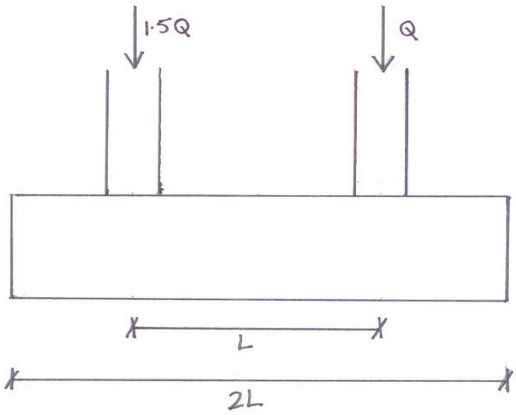
Then using notation $\lambda = \sqrt[4]{k / 4EI}$



• Considering a finite foundation like above:

$$\boxed{q(x) = \frac{2 K_s^2 B}{P\lambda} \cdot \frac{\sinh \lambda L + \lambda \sin \lambda L}{\cosh \lambda L + \cos \lambda L + 2}}$$

Example Design of a strip footing with column loading



1. Location of Resultant.

x = (1.5 QL) = 2.5Qx

$x = \frac{1.5QL}{2.5Q} \rightarrow \quad x = 0.6L_m$

2. Design of Base

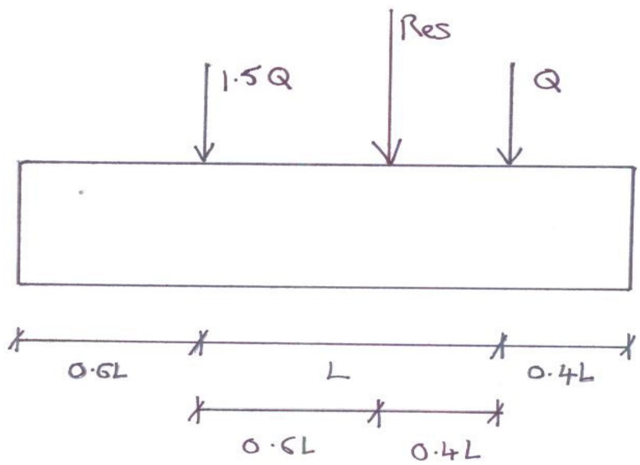
Take base to be of weight = $\frac{1}{2}$Q

Load = 0.5Q + 1.5Q + Q = 2Q KN

Bearing Capacity = ρ KN/m^2

Area Required = $\frac{2Q}{\rho}$ m^2

Area Provided = 2L x h = 2Lh m^2

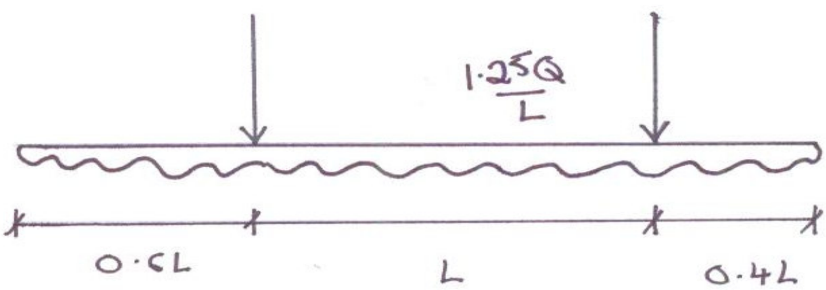


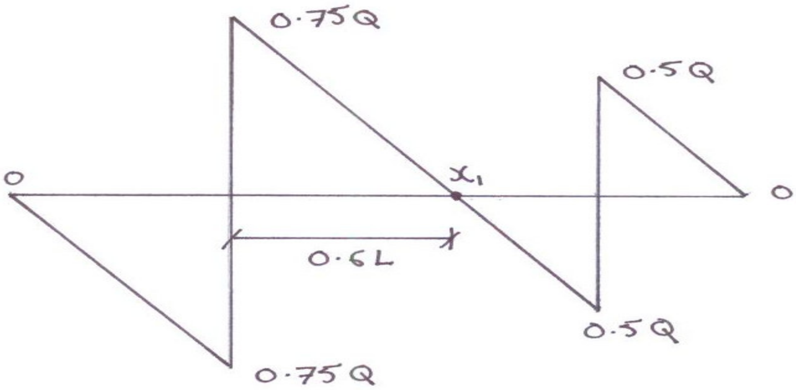
3. Ultimate Pressure

Pressure under the base of the foundation = $\frac{2.5Q}{2Lh} = \frac{1.25Q}{Lh}$

4. Design Reinforcement

UDL = $\frac{1.25Q}{Lh}$ x h = $\frac{1.25Q}{L}$





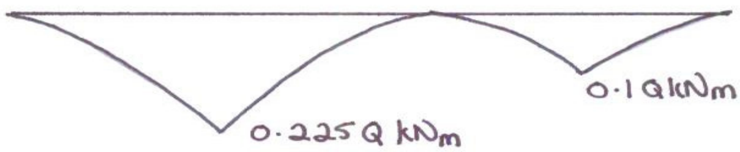
SFD

Zero shear at point x_1

$x_1 = \frac{0.75Q}{1.25Q/L} = 0.6L$

Max hogging moment at point of zero shear:

$1.5Q \times (0.6L) - \frac{(0.6L+0.6L)2}{2} \times \frac{1.25Q}{L} = 0$

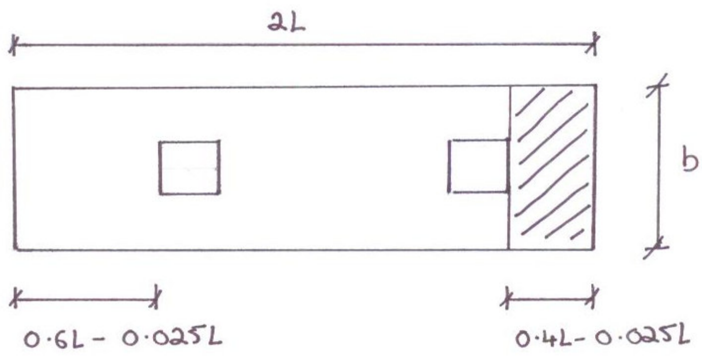


BMD

$k = \frac{M}{bd2Fcu} = 0$ .... ideal stiffness

As $k \leq 0.04 \rightarrow Z = 0.95d$

5. **Design for Shear**



Column width = 0.05L

Half column width = 0.025L

Direct shear at face of column = $\dfrac{\frac{1.25Q}{L}\ x\ b\ x\ 0.575L}{b\ x\ 0.95d}$

$= \dfrac{0.757Q}{d}$ must be $< 5.0\text{N/mm}^2$

$< 0.8\sqrt{Fcu}$

Direct shear at d from face of Column

0.6L – 0.95d – 0.025L = (0.575L – 0.95d)m

$= \dfrac{(0.575L - 0.95d)\,x\,b\,x\,1.25Q/L}{b\,x\,0.95d}$

$= \dfrac{0.757Q}{d} - \dfrac{1.25Q}{L} = Q\left(\dfrac{0.757}{d} - \dfrac{1.25}{L}\right)$

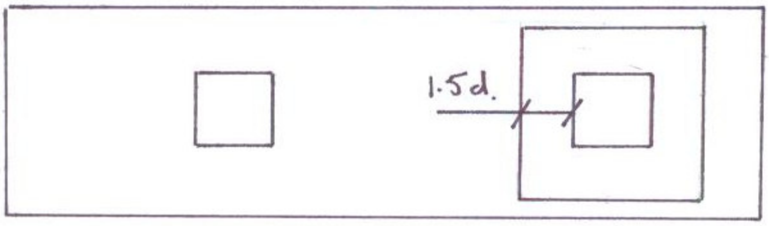
$Q\left(\dfrac{0.757}{d} - \dfrac{1.25}{L}\right) < 5.0\text{N/mm}^2$

$< 0.8\sqrt{Fcu}$

6. Punching Shear

$$@\ \frac{1.5Q}{4\ x\ 0.05\ x\ 0.95d} = \frac{7.9Q}{d} \quad < 5.0\text{N/mm}^2$$

$$< 0.8\sqrt{Fcu}$$



Conclusion

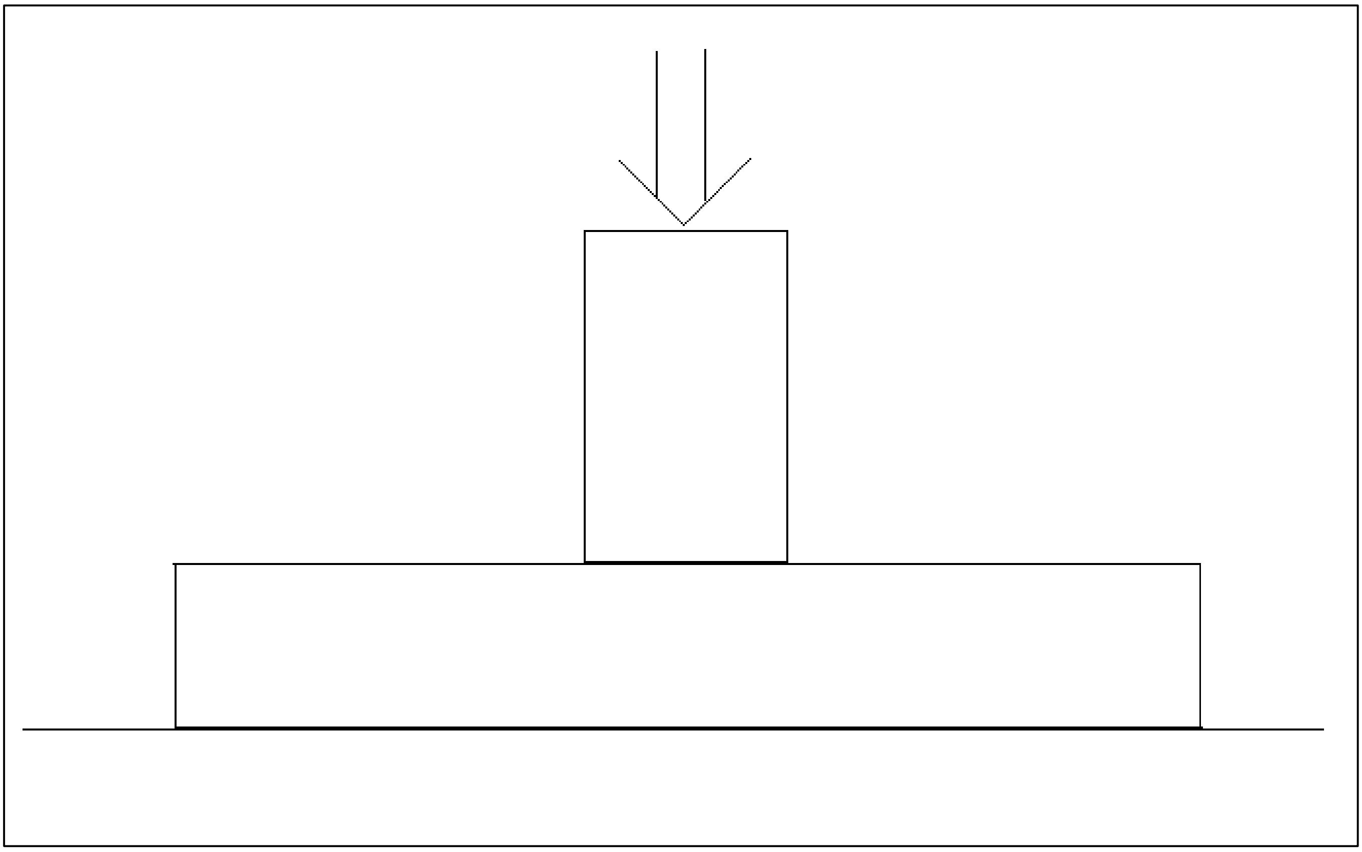
At the outset of this project, we didn’t fully understand the workings of a strip footings and its interaction with the soil below. We didn’t understand the relationship between the stiffness of the footing against the subgrade reaction of the soil and the variation of stress below the footing due to the column loads. However, after the completion of this assignment we now know that the stress below the footing depends on such factors as ; the spacing of the columns if two or more columns are on the footing, the depth and width of the footing, the distance from the point load to the point of stress and the stiffness of the footing. We found that the best equation to describe the stresses beneath the footing was Boussinesq’s formula. After determining that the stiffness would need to be infinite for the soil pressure to act as a uniform load across the bottom of the footing, we then analysed the problem as an upside-down continuous beam. For this criteria we derived an example of a footing using parameters to describe the various criteria required for the footing

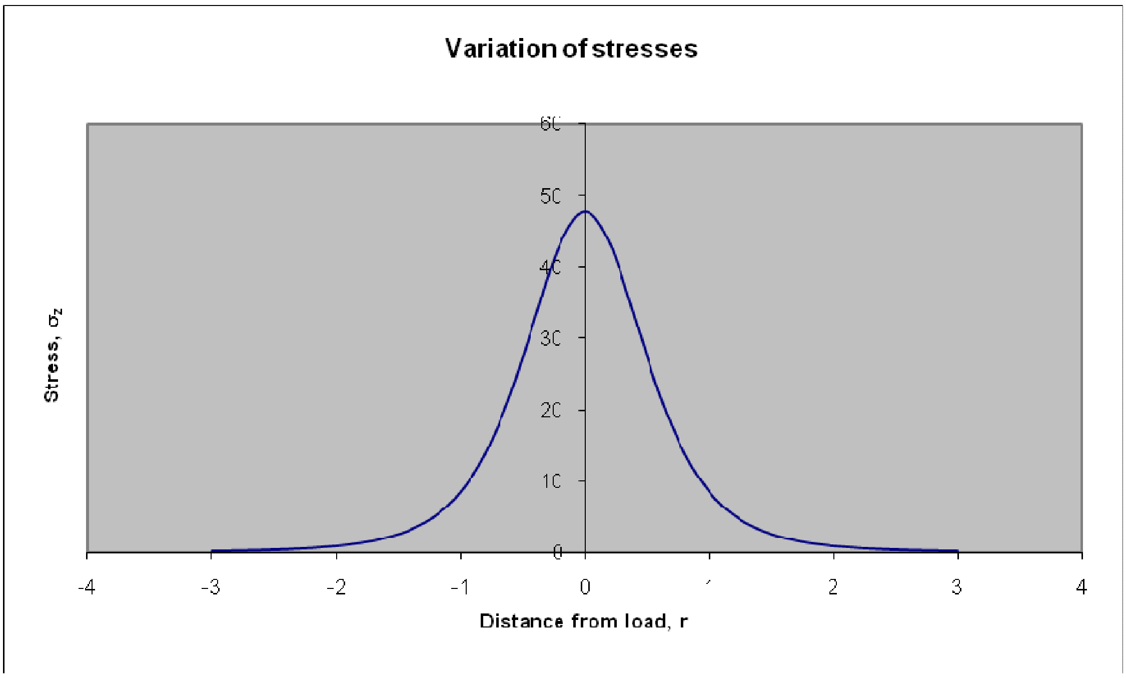
Excel Calculation

Stress distribution for a footing subjected to a column load, B= 6m & P= 100kN

P	100kN
z = depth of soil	1m
r = distance from the load	varies
σ_z =	$P/Z^2 * I_p$

r	z	r/z^2	Ip	σ_z
-3	1	9	0.0015	0.1511
-2.8	1	7.84	0.0021	0.2056
-2.6	1	6.76	0.0028	0.2848
-2.4	1	5.76	0.004	0.4021
-2.2	1	4.84	0.0058	0.5796
-2	1	4	0.0085	0.8545
-1.8	1	3.24	0.0129	1.2905
-1.6	1	2.56	0.02	1.9977
-1.4	1	1.96	0.0317	3.1691
-1.2	1	1.44	0.0514	5.1367
-1	1	1	0.0844	8.4447
-0.8	1	0.64	0.1387	13.869
-0.6	1	0.36	0.2215	22.147
-0.4	1	0.16	0.3296	32.962
-0.2	1	0.04	0.4331	43.309
0	1	0	0.4777	47.771
0.2	1	0.04	0.4331	43.309
0.4	1	0.16	0.3296	32.962
0.6	1	0.36	0.2215	22.147
0.8	1	0.64	0.1387	13.869
1	1	1	0.0844	8.4447
1.2	1	1.44	0.0514	5.1367
1.4	1	1.96	0.0317	3.1691
1.6	1	2.56	0.02	1.9977
1.8	1	3.24	0.0129	1.2905
2	1	4	0.0085	0.8545
2.2	1	4.84	0.0058	0.5796
2.4	1	5.76	0.004	0.4021
2.6	1	6.76	0.0028	0.2848
2.8	1	7.84	0.0021	0.2056
3	1	9	0.0015	0.1511





Variation of stresses
Stress, σ_z
Distance from load, r
60
50
40
30
20
10
0
-4
-3
-2
-1
0
2
3
4

References

Jaroslav F. *Stress in subsoil and methods of final settlement calculation*, Publishing house of the Czechoslovak Academy of Sciences, Prague, 1978.

Whitlow R. *Basic Soil Mechanics*, Pearson Education Limited, England, 2001

Sowers B. *Introductory soil mechanics and Foundations*, MacMillan Publishing, New York, 1989.

Cernica, J.N., *Geotechnical Engineering: Foundation Design*, John Wiley & Sons, Inc., New York, 1995.

Das, B.M., *Principles of Foundation Engineering*, 4TH Edition, PWS Publishing, 1999, USA.