Structural Analysis Lab Submission 4

Linear Analysis of RC beam

DT024/4

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Introduction

Nonlinear analysis is the analysis of a section outside its linear range. This means analysing a structure when it no longer obeys Hooke's Law, stress is no longer proportional to strain. This is known as designing a structure in its plastic region. This allows a structure to take more load as the material properties have initial yield stress values but can take more up to an ultimate yield state. Only non brittle materials can be designed this way, therefore in this report the nonlinear material is the steel reinforcement in the concrete section.

In this report the behaviour of a simply supported beam subjected to two separate loading conditions shall be investigated. The first of these loading conditions is a point load located at the beams mid-span and the other being a uniformly distributed load. The objective of the report is to compare a non-linear analysis of the beam to theoretical calculations. A finite element analysis programme shall be used to predict the non-linear behaviour.

The beam was designed as part of a six meter grid in a concrete frame for an office building, imposed load of 5kN/m2. This beam was then analysed with a point load.

Finite Element Analysis: Point Load

Steel

Steel Ult Yield 500

Steel Initial Yield 300

E = 210e3 N/mm2

Poisson's Ratio = 0.3

Concrete

Yield 35N/mm

E = 42000

Poisson's Ratio = 0.2

Comments:

Not enough nodes for UDL

Need to set Design Strengths (Not done in Manual)

Modelling in 2D is fine, no need for 3D

Not enough Nodes so only half of each beam modelled

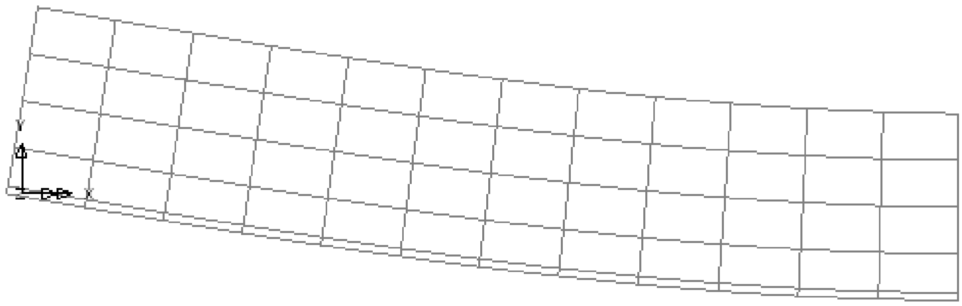


Fig.1 Deflected shape

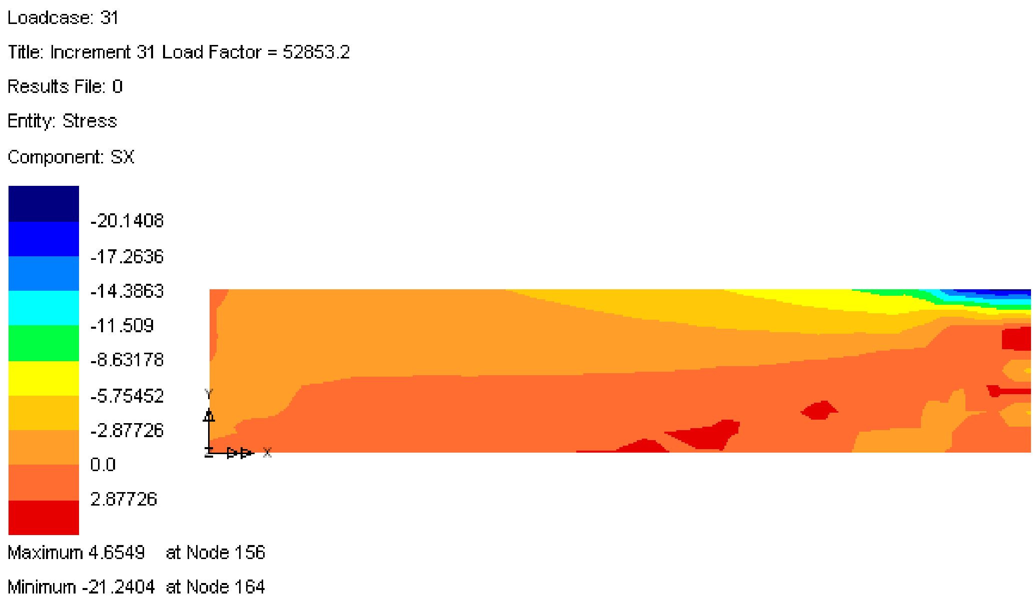


Fig.2 Stress in beam at failure

From Fig. 2 we can see that the max stress distribution in the concrete is located at the top third of the section. This shows that the concrete section is capable of taking more load, as if it were fully loaded the max stresses theoretically should stretch down to the neutral axis.

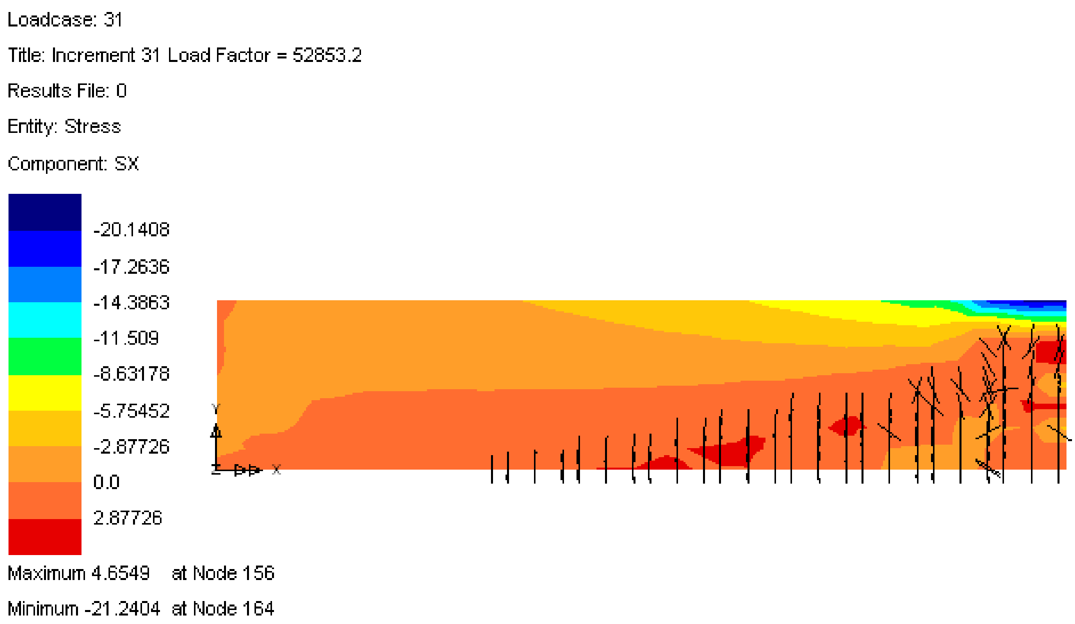


Fig.3 Cracks in beam at failure, 52.853kN

Fig.3 shows that the concrete is cracking wherever a tensile stress is exerted on it.

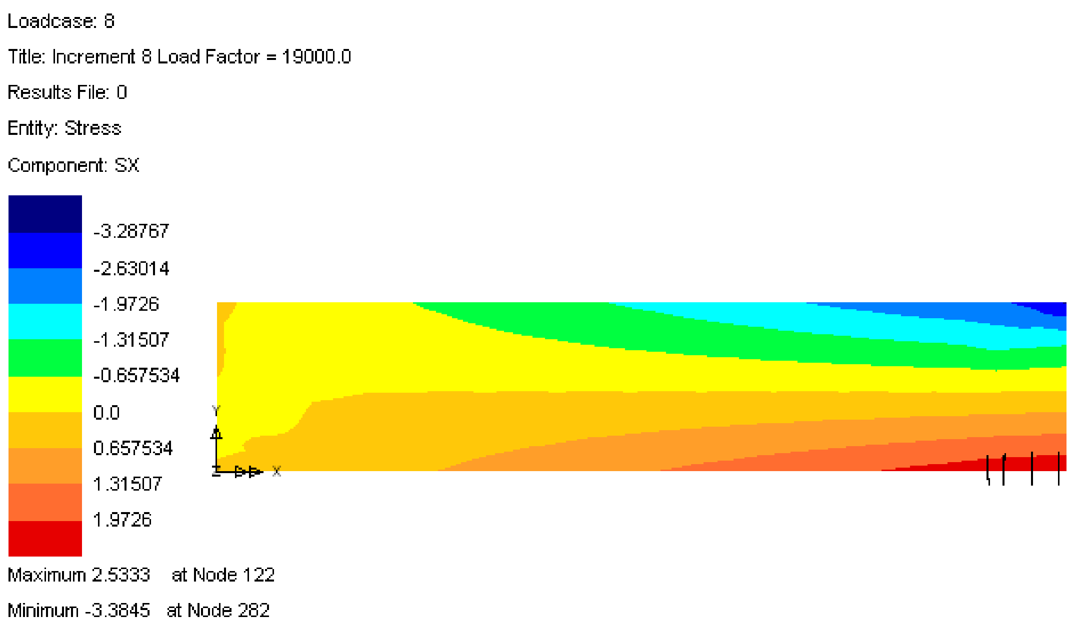
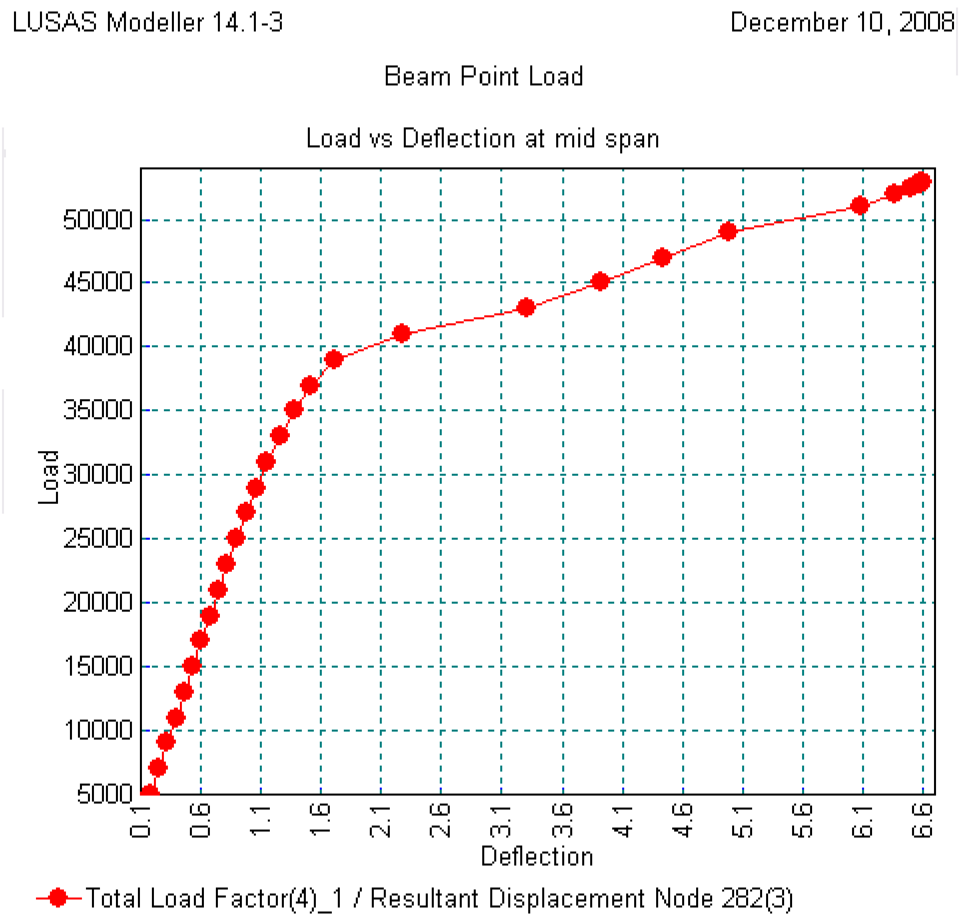
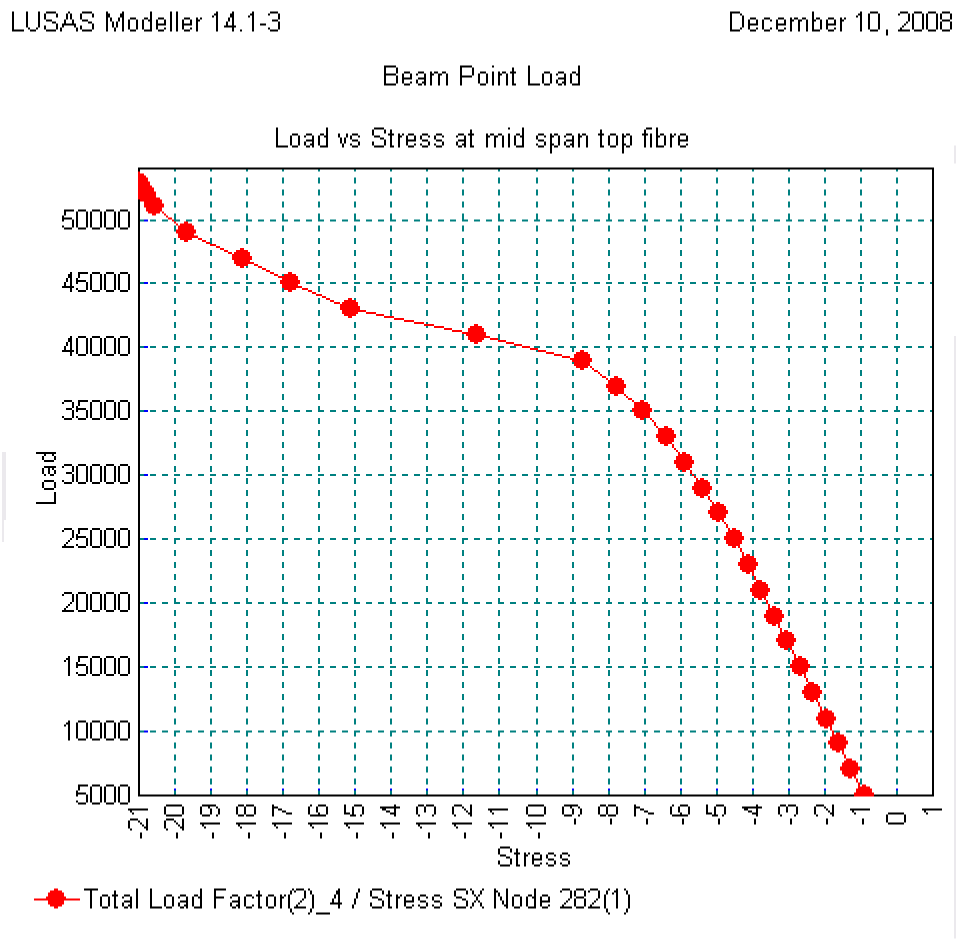


Fig.4 First cracks appear at 19 kN



It can be seen from the graph that there is a linear relationship between load and deflection up until a load of around 37kN. This range shows that up until this load the section was linearly elastic. It is clear that the section then behaves plastically, but is not perfectly plastic as the section still takes more load increments. This is due to the initial yield value in the steel of $300N/mm^2$ and an ultimate stress value of $500N/mm^2$



The load vs stress graph confirms the same region of linear elasticity, up until a load of 37kN, and exhibits the same plastic behaviour, as expected.

Finite Element Analysis: Uniformly Distributed Load

The LUSAS model of the UDL was approximated to a point load located at one meter intervals along the length of the beam. This was done as the student version of LUSAS was unable to cope with a full UDL.

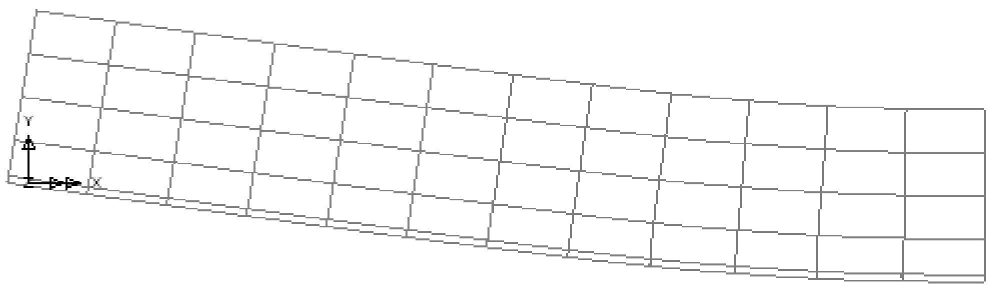


Fig.5 Deflected Shape

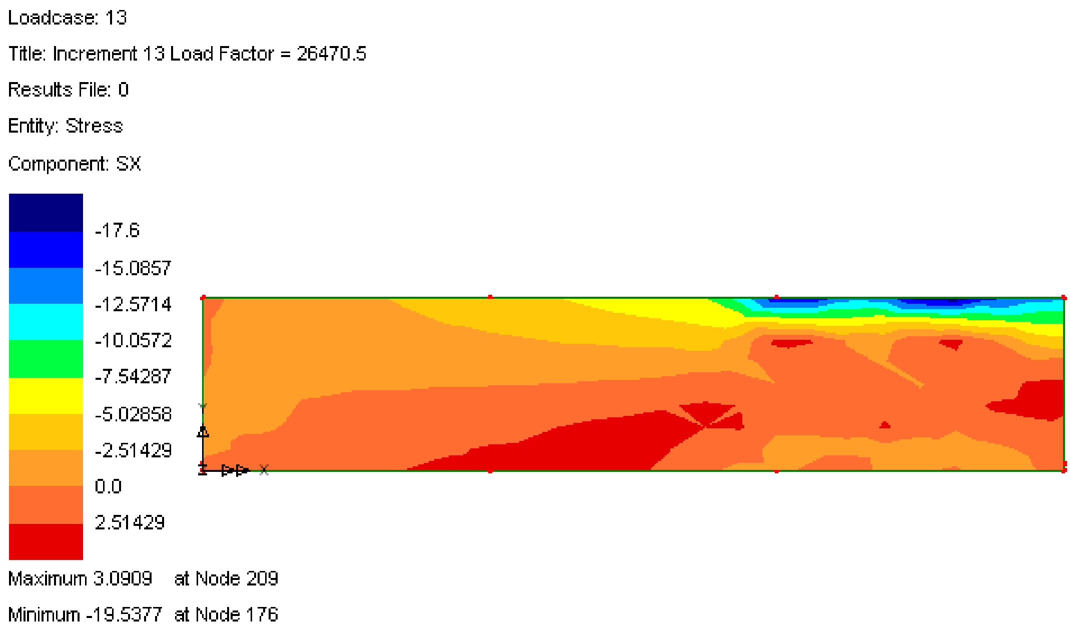


Fig.6 Stress Distribution at failure, 26.052kN

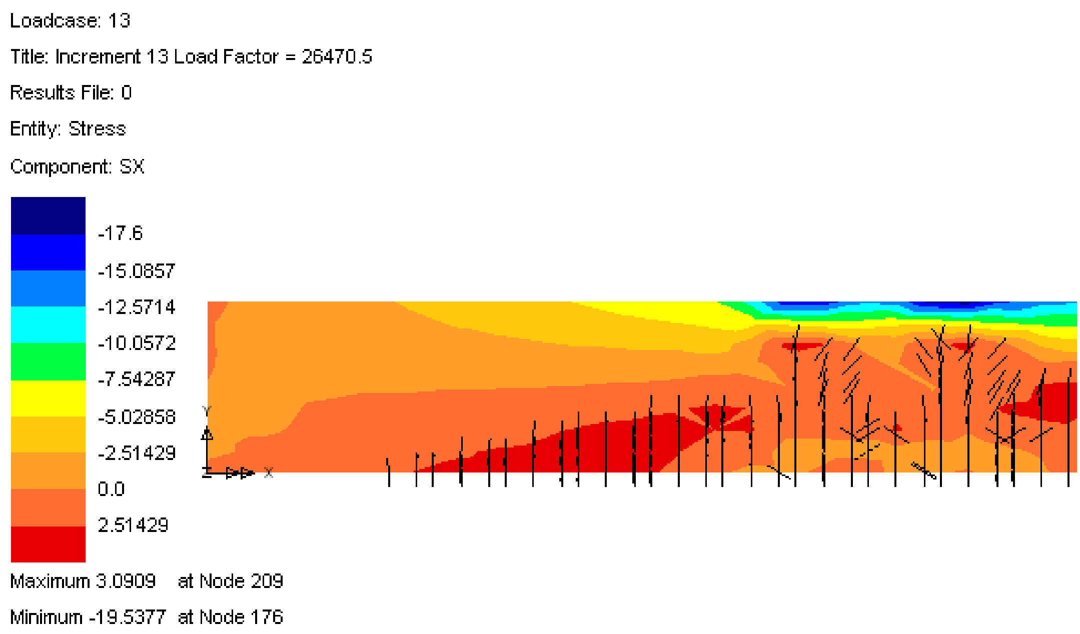


Fig.7 Crack pattern at failure

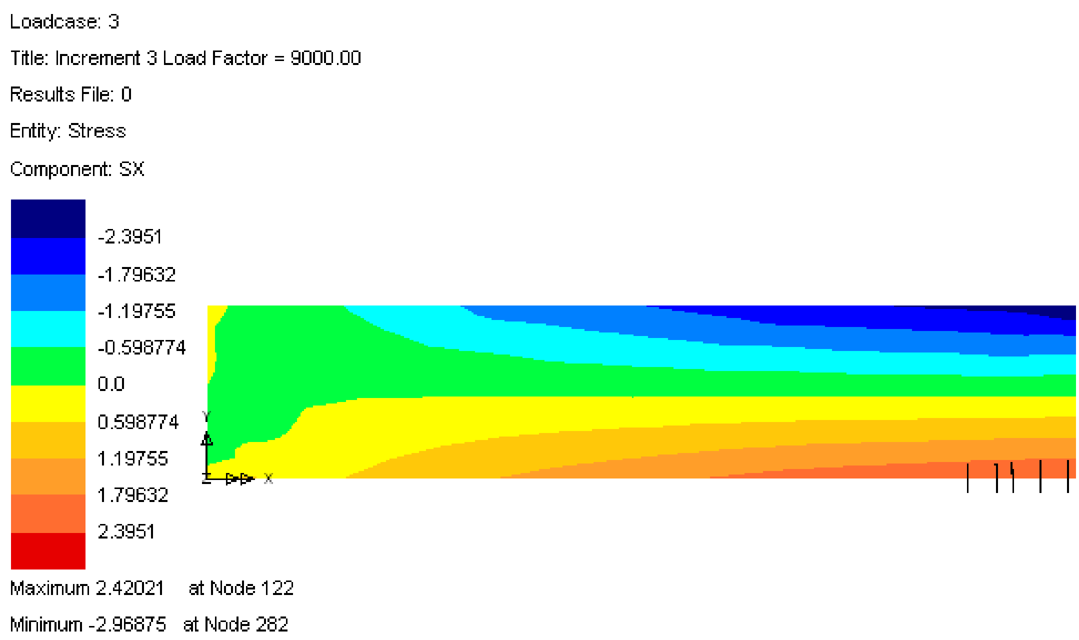
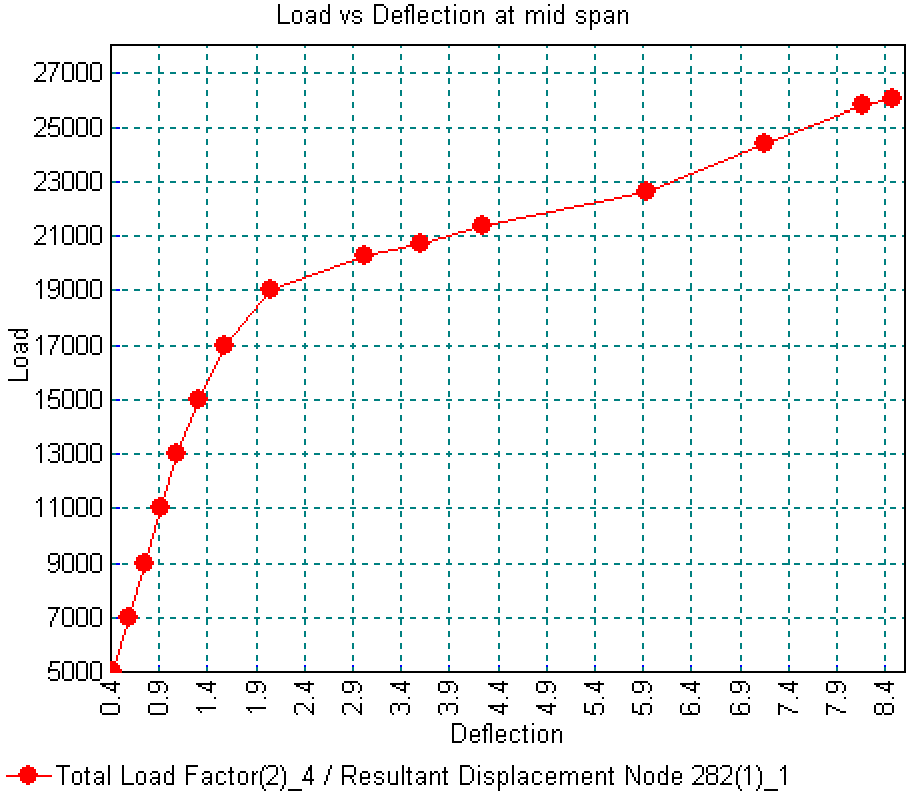


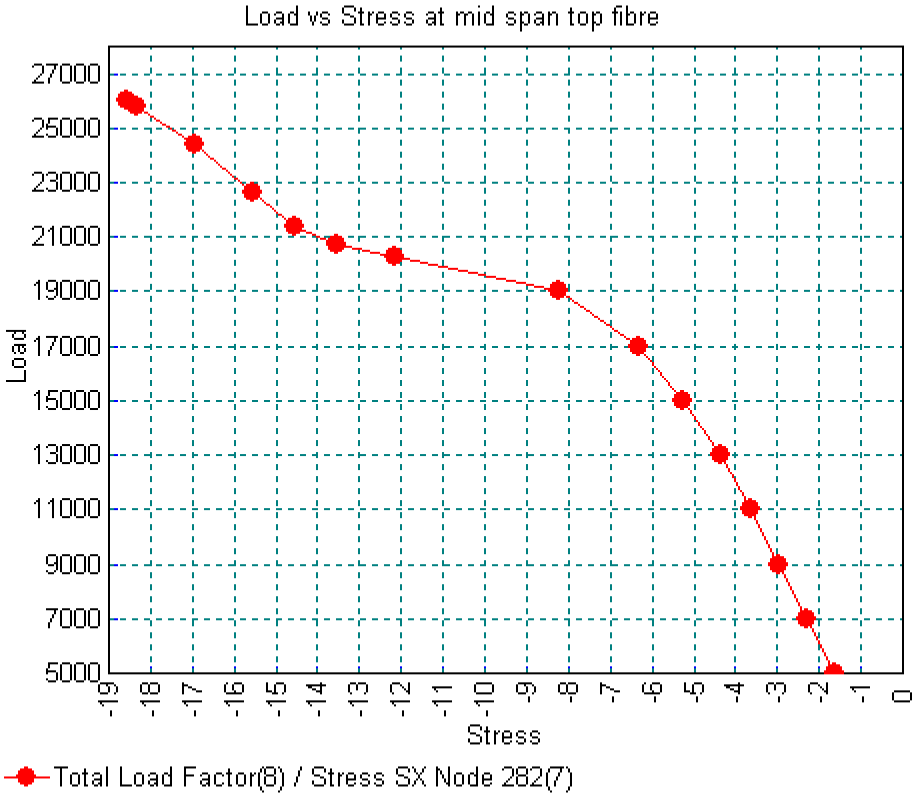
Fig.8 First cracks 9kN

Beam Point Load



LUSAS Modeller 14.1-3 December 10, 2008

Beam Point Load



Modelling

1. In this case only half the beam was modelled. It was only possible to do this because the beam is symmetrical and loaded at mid-span. It was also only possible to model half the beam in the student version of LUSAS as there was not enough nodes in this version to model the full beam
2. At the support a horizontal roller was used to prop the vertical direction, as if a pinned support was used (restraining the vertical and horizontal directions) yielding of the concrete occurs in the concrete at the supports causing an early failure of the beam. As only half the beam is being considered vertical rollers must be placed at the end of the beam, i.e. at the mid-span of the actual beam.
3. When considering the beam under a uniformly distributed load (UDL), again there wasn't enough nodes in the student version, so point loads where applied at 1m spacing to try simulate a UDL.
4. In this case the mesh density was set at a constant value throughout the beam. An ideal mesh density was not examined.
5. If failure occurs in the model, the manual doesn't really help as it is just design examples.

Results

By carrying out a theoretical analysis (Appendix A and B) and by modelling (In LUSAS) symmetrically similar concrete beams it was possible to predict the failure loads, shown in table 1.

Table 1:

	Theoretical Result (kN)	LUSAS Model Results (kN)	% Difference
Point Load	99.926	105.706	5.50%
UDL	33.31	26.052	21.80%

- For the case of the beam subjected to a single point load at mid-span, the failure load calculated theoretically is 5.50% smaller than the value predicted from modelling the beam in LUSAS. This percentage difference may be due to assumptions which are made during hand calculations (i.e. simplified stress block). Noting that although the value calculated analytically may be slightly conservative it is still an accurate method of calculating the failure load.
- In the second case where the beam was subject to a UDL we find that there is a larger percentage difference and the theoretical value is larger than the LUSAS models result. As we were unable to model the UDL due to limitations in the allowable number of nodes in the student version of LUSAS we had to model it as a series of point loads at 1m spacing. These point loads lead to the beam failing earlier than if the load had have been evenly distributed over the length of the beam.

Conclusion

For the case of the beam subjected to a point load at mid-span we were able to predict the failure load by hand calculations and by use of a finite element analysis, the values achieved were reasonably similar with the hand calculations being slightly conservative. As we were unable to efficiently model the case of the beam being subjected to a uniformly distributed load we can't really draw any decisive conclusions for this loading. The finite element analysis programme would be more useful if you have the full package, as then it could be used for more unusual loading patterns and support conditions, to determine the exact locations of maximum stresses to accurately design the section and place reinforcement.

References

Bhatt, Prab, MacGinley, Thomas J, Choo, Ban Seng. 2007. Reinforced *Concrete Design Theory and Examples*. 3rd Ed. Taylor and Francis: London and New York

Gere, JM. 2004. *Mechanics of Materials*. 5th Ed. Thomson Business Information

Maekawa, K, Pimanmas, A, Okamura, H. 2003. *Nonlinear Mechanics of Reinforced Concrete*. Spon Press: London and New York

Appendix A: Simply Supported Beam Subjected to a UDL

Assume: Beam Width = 300mm

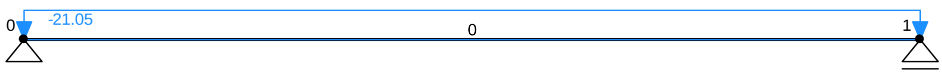
Beam Depth = 600mm

Loading:

Dead Load = Self-weight of beam:	(0.3)(0.6)(24)	= 4.32kN/m	
		= 5.0kN/m	
	Total Dead Load	= 9.32kN/m	
	Total Live Load		= 5.0kN/m

Design Load: (1.4)(9.32) + (1.6)(5.0) = 21.05kN/m

6m Span:

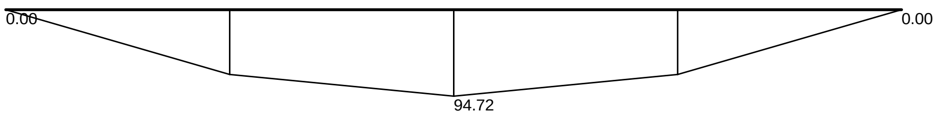


Structural Design:

Effective Depth, d : 600-25-12-10 = 553mm

Maximum Moment, M_{max} $= \frac{wl^2}{8}$ = (21.05)(6.0^2)/(8) = 94.73kNm

Bending Moment Diagram:



k = $(94.73\text{x}10^6)/(300)(553^2)(35)$ = 0.030

Note: Since k is less than 0.156 (Given in the codes) there is no compression steel required

Lever arm: $z = (553)(0.5 + \sqrt{0.25} - 0.03/0.9)$ = 534mm

As Required: $(94.73\text{x}10^6)/(0.95)(500)(535)(0.95)$ = 393mm^2

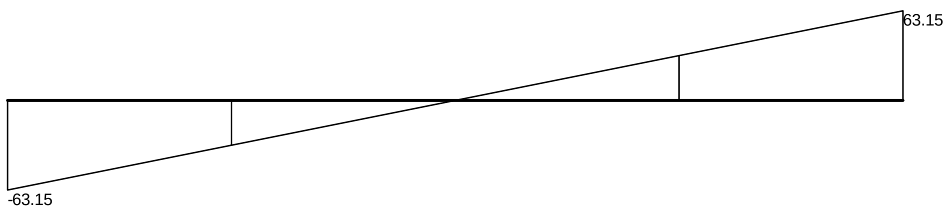
As = $\frac{(0.13)}{(100)}(1000)(600)$ = 780mm^2

Therefore use 4T16's providing 804 mm^2

<u>Max Shear force:</u>

Max shear force = (21.05)(6.0)/(2.0) = 63.15kN

Shear Force Diagram:



v = (63.15x103)/(300)(553) =0.381N/mm^2

$v_c = (0.632)\left[\frac{((100)(393))}{((1000)(553))}\right]^{\frac{1}{3}}(1.0)(1.12)$

= 0.293 N/mm^2

$v_c < v$ Therefore Shear is not a problem

<u>Spacing:</u>

$Sv = \dfrac{(0.95)(500)(Asv)}{(553)(0.381 - 0.293)}$ = 9.76 *Asv* if we use 2 links of T12's

Sv = (553)(9.76) = 5397.3mm c/c

But max c/c =0.75d =(0.75)(553) = 414.75mm^2

Therefore provide 2T12 @ 400 c/c links

<u>Deflection:</u>

Simply supported: Span/depth =20

Service Stress: $fs = \dfrac{(2)(500)(393)}{(3)(804)}$ = 163

Modification factor: $\dfrac{0.55 + (477 - 163)}{120(1.03 + 0.9)}$ ≤ 2.0

= 1.358 ≤ 2.0

Allowable Deflection = (1.358)(20) = 27.16

Actual Deflection = (6000)/(553) =10.85

<u>Maximum UDL that can be applied to the beam:</u>

$M_{max} = \dfrac{wl^2}{8} = Mr = 149.89$

$M_{max} = Mr$

→ w = (149.89)(8)/(6^2) = <u>33.31kN/m</u>

Appendix B: Point Load at Mid-Span

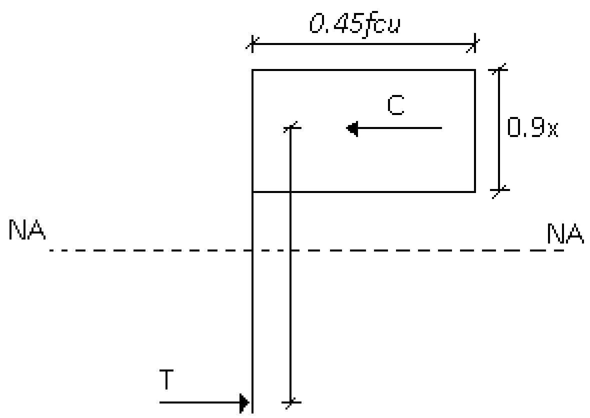
Note: Use the same beam dimensions and steel reinforcement as Appendix A

Moment Capacity of Section:

fcu = 35N/mm^2

fy = 500N/mm^2

Concrete Stress Block:



From Appendix A, As = 4T16 = 804 mm^2

<u>Tension force of steel:</u> $0.87fyAs$ = (0.87)(500)(804) =349.74kN

$x = 0.5d$ = (0.5)(553) = 276.5mm

<u>Compressive Force of Concrete:</u> $0.45fcu(0.9xb)$

= (0.45)(35)(0.9)(0.5)(553)(300) = 1175.82kN

$Z = d - 0.45x \quad = d\text{-}0.45(0.5d) \quad = 0.775d \quad = 0.775(553) \quad = 428.58\text{mm}$

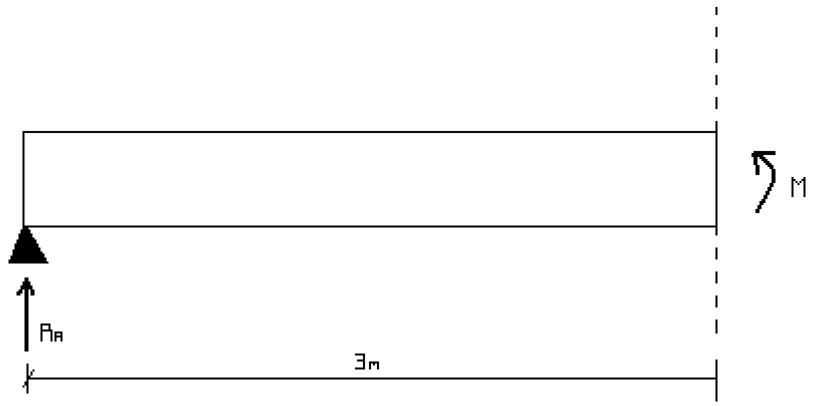
<u>Moment resistance of section:</u> Mr = Tension in steel by lever arm

$$Mr = (349.74)(428.58) = \underline{149.89\text{kNm}}$$

Note: This is the maximum moment the beam can take, any value above this and the beam will have a tension failure in the rebar

To convert this into a point load we can just work backwards from simple beam analysis:

Cut the beam at mid-span:



Given that the beam is symmetrical and the point load is at mid-span we can say that the reactions are equal and the maximum moment occurs at mid-span.

Let $M_{max} = Mr = \underline{149.89\text{kNm}}$

Taking moments about the mid-span: $M_{max} - R_A(3\text{m}) = 0$

$$149.89 - 3R_A = 0$$

$$R_A = 149.89/3 = 49.963\text{kN} = R_B$$

Therefore the maximum point load that can be applied to the beam is equal to $R_A + R_B$

$$P_{max} = \underline{99.926\text{kN}}$$