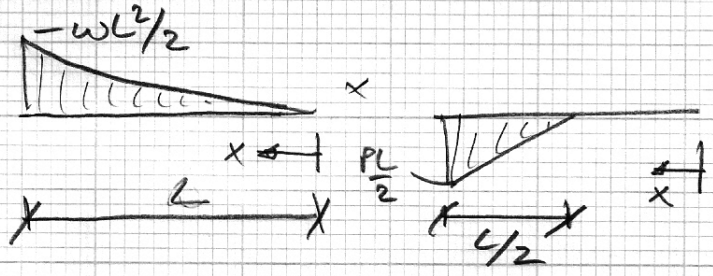
Integration of Part-wise



$M(x) = wx^2/2$ $\quad$ $\delta M(x) = P(x - L/2)$ for $L/2 \le x \le L$

$$\Rightarrow \int M(x) \cdot \delta M(x)\, dx = \int_{L/2}^{L} \frac{wx^2}{2} \cdot P \cdot (x - L/2)\, dx$$

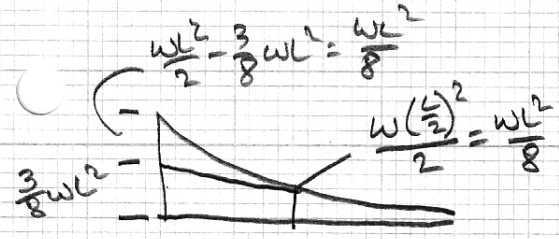
$$= \frac{Pw}{2} \int_{L/2}^{L} \left(x^3 - x^2 \cdot \frac{L}{2}\right) dx$$

$$= \frac{Pw}{2} \left[\frac{x^4}{4}\Big|_{L/2}^{L} - \frac{L}{2} \cdot \frac{x^3}{3}\Big|_{L/2}^{L} \right]$$

$$= \frac{Pw}{2} \left[\left(\frac{L^4}{4} - \frac{L^4}{64}\right) - \frac{L}{6}\left(L^3 - \frac{L^3}{8}\right) \right]$$

$$= \frac{Pw}{2} \left[\frac{15L^4}{64} - \frac{7L^4}{48} \right]$$

$$= \underline{\frac{17}{384} \cdot PwL^4} \quad \leftarrow \text{Exact answer}$$



M@ support due to WL on $x \le L/2$

$= \left(w \cdot \frac{L}{2}\right)\left(\frac{3}{4}L\right) = 3/8\, wL^2$

Areas: Trapezoid: $\frac{1}{6}\left(\frac{PL}{2}\right)\left(\frac{wL^2}{8} + 2 \cdot \frac{3}{8}wL^2\right)\left(\frac{L}{2}\right) = \frac{7}{192} PwL^4$

Parabola: $\frac{1}{4}\left(\frac{PL}{2}\right)\left(\frac{wL^2}{8}\right)\left(\frac{L}{2}\right) = \underline{\frac{1}{128} PwL^4}$

Table formulae ↗

Total $= \underline{\underline{\frac{17}{384} PwL^4}}$