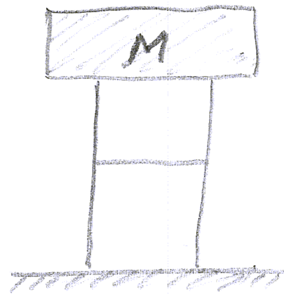
EXAMPLE 2.1

An harmonic oscillation test gave the natural frequency of the water tower to be 0.41Hz. Given that the mass of the tank is 150 tonnes, what deflection will result if a 50kN horizontal load is applied. You may neglect the mass of the tower structure



$f = 0.41\,Hz$

$M = 150$ tonnes

$$\omega = 2\pi f = 2\pi \times 0.41 = 2.576 \text{ rad/s}$$

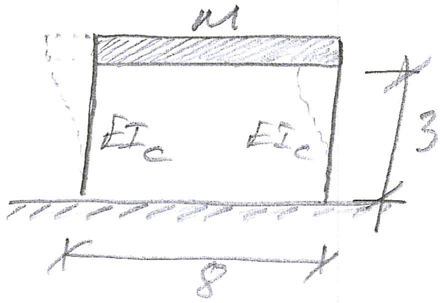
but $\omega = \sqrt{k/m}$ $\therefore$ $k = m \cdot \omega^2 = 150 \times 10^3 \times 2.576^2$

$$= 995 \text{ kN/m}$$

$\therefore$ $\delta = 50/995 = 0.0502\,m = \underline{50.2\,mm.}$

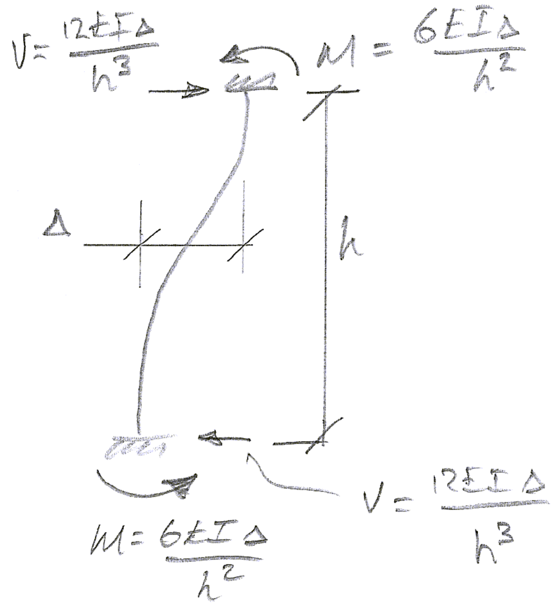
Example 2.2

The frame shown is rigidly jointed. You may ignore the mass of the columns. Calculate the natural frequency in lateral vibration and its period. Find the force required to deflect the frame 25mm laterally.



$m = 5000\,kg$

$EI_c = 4.5 \times 10^3\ kN\,m^2$



from the figure, we see:

$$\Delta = F/k \quad \therefore\ k = F/\Delta$$

$$\therefore\ k = \frac{12EI\Delta}{h^3} \Big/ \Delta = \frac{12EI}{h^3}$$

For our frame, there are two columns

$$\therefore\ k = 2 \times \frac{12EI_c}{h^3}$$

$$= 2 \times \frac{12(4.5 \times 10^6)}{3^3}$$

$$= 4.0 \times 10^6\ N/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4 \times 10^6}{5 \times 10^3}}$$

$$= \underline{4.502\ Hz} \qquad \therefore\ T = 1/f = \underline{0.2221\ secs}$$

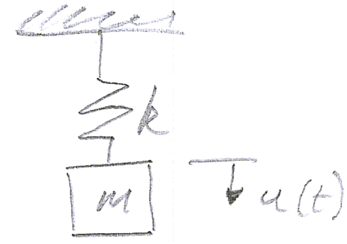
When $\delta = 25\,mm$, we have, $F = k\delta = 4 \times 10^6 \times 0.025/10^3$

$$\therefore\ \underline{F = 100\,kN.}$$

Example 2.3

The mass of fig 1 is given an initial displacement of 10mm and velocity of 100mm/s.

1) Find the natural frequency
2) Find the period of vibration
3) Find the amplitude of vibration
4) What time does the 3rd maximum peak occur at?



$m = 20kg$

$k = 350 N/m$

$u_0 = 10mm$

$\dot{u}_0 = 100 mm/s$

Fig 1

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1) $\omega = \sqrt{\frac{k}{m}} = \sqrt{350/20} = 4.1833 \ rad/s$

$\omega = 2\pi f \ \therefore f = \omega/2\pi = 4.1833/2\pi = \underline{0.666 \ Hz}$

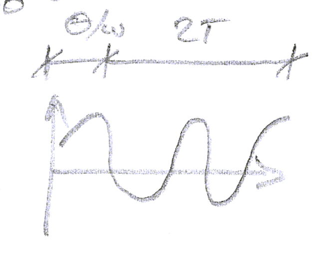
2) $f = \frac{1}{T} \ \therefore T = 1/f = 1/0.666 = \underline{1.502 \ secs.}$

3) $\rho = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega}\right)^2} = \left[10^2 + \left(\frac{100}{4.1833}\right)^2\right]^{0.5} = \underline{25.91mm}$

4) The time to the first maximum is:

$t_{1,max} = \theta/\omega$ where $\theta = \tan^{-1}\frac{\dot{u}_0}{u_0\omega}$

$\therefore \theta = \tan^{-1}\frac{100}{10 \times 4.1833} = 1.1745 \ rads$



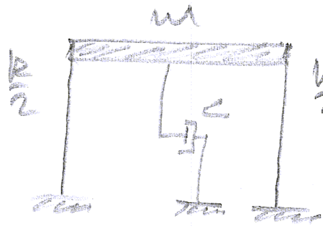
$\therefore t_{1,max} = 1.1745/4.1833 = 0.2807 secs.$

Thereafter, peaks occur at T secs

$\therefore$ Time to 3rd $= 0.2807 + 2 \times 1.502 = \underline{3.285 \ secs.}$

Example 2.4

For the frame of Ex. 2.2, a jack applied 100kN and then instantaneously released. On the first return swing a deflection of 19.44 mm was noted. The period of motion measured was 0.223 secs. Assuming that the stiffness of the columns is as Ex. 2.2, find



1) The effective weight of the girder
2) The damping ratio, ξ
3) The coefficient of damping, c
4) The undamped frequency and period
5) The amplitude after 5 cycles.

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1) $T_d = 0.223 = \frac{2\pi}{\omega_d} \approx 2\pi\sqrt{\frac{m}{k}}$

$\therefore m = \left(\frac{0.223}{2\pi}\right)^2 \cdot k = \left(\frac{0.223}{2\pi}\right)^2 \times 4\times10^6 = \underline{5{,}039 \text{ kg}}$

2) logarithmic decrement, $\delta = \ln \frac{u_n}{u_{n+1}}$

where $u_n = \frac{100\times10^3}{4\times10^6} = 0.025\text{m} = 25\text{mm}$

$\therefore \delta = \ln \frac{25}{19.44} = 0.2515$

but $\xi \approx \frac{\delta}{2\pi}$ $\therefore \xi = \frac{0.2515}{2\pi} = \underline{0.04}$

i.e. 4% of critical

3) $\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{0.223} = 28.1757 \text{ rad/s}$

but $\omega_d = \omega\sqrt{1-\xi^2} \therefore \omega = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{28.1757}{\sqrt{1-0.04^2}}$

$\therefore \omega = 28.198 \text{ rad/s}$

$c_{cr} = 2m\omega \quad \therefore c = \xi \cdot 2m\omega$

$= 0.04 \times 2 \times 5039 \times 28.198$

$= \underline{11367 \text{ kg}\cdot\text{s/m}}$

4) $\omega = 2\pi f \quad \therefore f = \omega/2\pi = \frac{28.198}{2\pi} = \underline{4.488 \text{ Hz}}$

$T = 1/f = 1/4.488 = \underline{0.2228 \text{ secs.}}$

5) $u_{n+p} = \left(\frac{u_{n+1}}{u_n}\right)^p u_n$

$\therefore u_5 = \left(\frac{19.44}{25}\right)^5 \cdot 25 = \underline{7.11 \text{ mm}}$

Example 2.5

For the response time history given,

1) Estimate the damped natural frequency
2) Use the half-amplitude method to estimate the damping, ξ
3) Calculate the undamped natural frequency and period.

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1) Scaling from the figure, the distance from the 2nd to 5th peak is 119mm
$\rightarrow$ 3 full cycles
But 2secs measures 178 mm

$\therefore$ 3cycles $= \frac{119}{178} \times 2 = 1.337$ secs

$\therefore T_d = 1.337/3 = \underline{0.4456 \text{ secs}}$.

$\therefore f_d = 1/T_d = \underline{2.24 \text{ Hz}}$

2) 50mm displacement $\rightarrow$ 80mm on page.
1st peak $\rightarrow$ 67mm i.e. 41.9mm disp
3rd peak $\rightarrow$ 36mm i.e. 22.5mm disp
} 2cycles

$\therefore$ half amplitude takes $\frac{22.5/41.9}{0.5} \times 2 = 2.148$ cycles

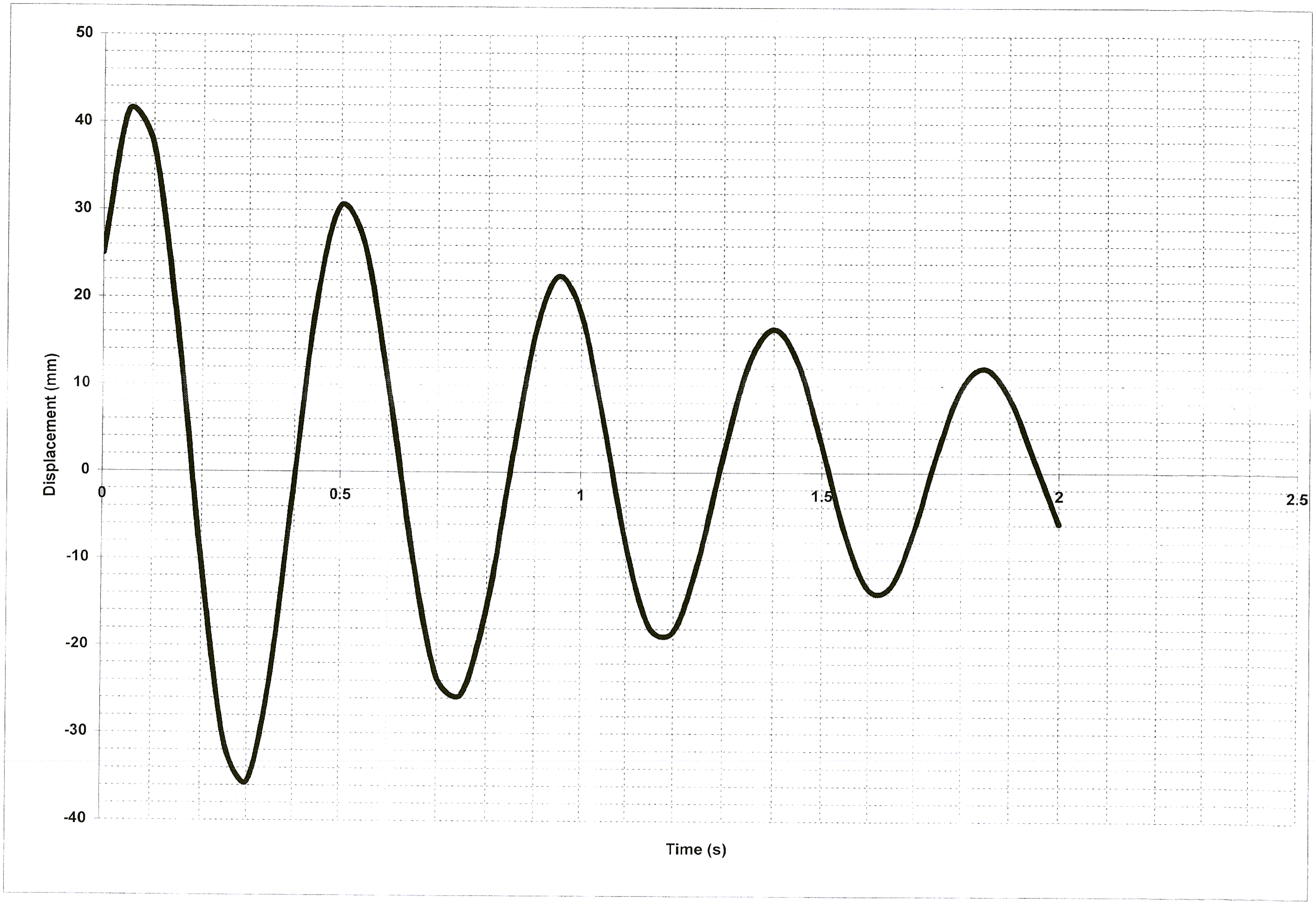
$\therefore \xi = \frac{0.11}{2.148} = \underline{0.0512}$ (0.05 actually)

3) $\omega/\omega_d = \sqrt{1-\xi^2}$ $\therefore \omega = 2\pi(2.24)\sqrt{1-\xi^2}$
$= 14.05$ rad/s

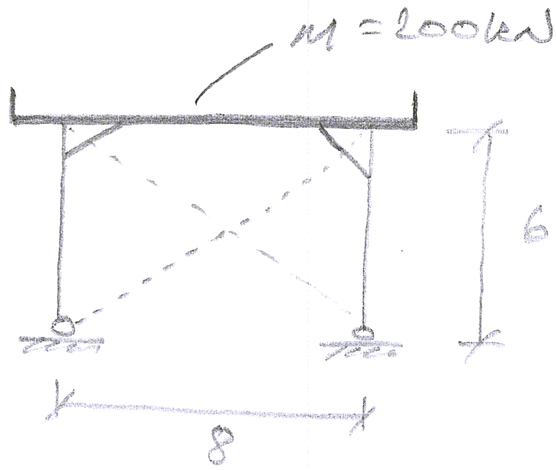
$\therefore f = \omega/2\pi = \underline{2.236 \text{ Hz}}$

$\therefore T = 1/f = \underline{0.447 \text{ secs}}$

Example 2.5



Example 2.6



The owner of a working platform has complained to the engineer because the workers' movement is causing large dynamic motion. The engineer investigated and found the natural period in sway to be 0.9 sec. Remedial ties (shown dotted) are to be installed to reduce the natural period to 0.3 sec. Given $E = 200\,kN/mm^2$, what diameter of tie is required?

Example 2.6

$f = 1/T = 1/0.9 = 1.111\ Hz$

but $f = \frac{1}{2\pi}\sqrt{k/m} \Rightarrow k = (2\pi f)^2 \cdot m$

$\therefore k_{now} = \left(2\pi \times \frac{1}{0.9}\right)^2 \times (200\times10^3/9.81)$ → mass in kg $= 20{,}387\ kg$

$= 993.655\ kN/m$

$k_{new} = \left(2\pi \times \frac{1}{0.3}\right)^2 \times (20{,}387)$

$= 8{,}942.896\ kN/m$

$\therefore k_{cable} = k_{new} - k_{now}$

$= 7{,}949.24\ kN/m.$

From the attached sheet,

$$k_{cable} = \frac{EA}{L}\cos^2\alpha = EA\frac{L_x^2}{(L_x^2 + L_y^2)^{1.5}}$$

$$= EA \cdot \frac{8^2}{(8^2 + 6^2)^{1.5}}$$

$$7{,}950 = EA \times 0.064$$

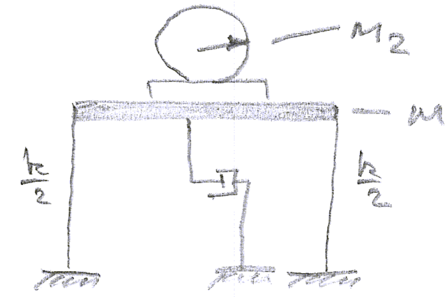
$$\therefore EA = 124{,}206\ kN$$

$$\therefore A = 124{,}206/200$$

$$= 621\ mm^2$$

i.e. $28.1\ mm\ \phi$

Example 2.7



$m_1 = 5{,}000$ kg
$m_2 = 4{,}000$ kg
$k = 4\times10^6$ N/m

The frame of examples 2.2 and 2.4 has a reciprocating machine put on it. The mass of this is to be allowed for. The machine exerts a periodic force of 8.5 kN at a frequency of 1.75 Hz.

a) What is the steady-state amplitude of vibration if $\xi = 4\%$?

b) What would the steady-state amplitude be if the forcing frequency was in resonance with the structure?

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a) The static deflection is, $u_{st} = \frac{F_0}{k} = \frac{8500}{4\times10^6} = 2.125\text{mm}$

The mass is now 9,000 kg

$$\therefore f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{4\times10^6/9{,}000} = 3.355\,\text{Hz}$$

The frequency ratio $\beta = \frac{\Omega}{\omega} = \frac{1.75}{3.355} = 0.522$

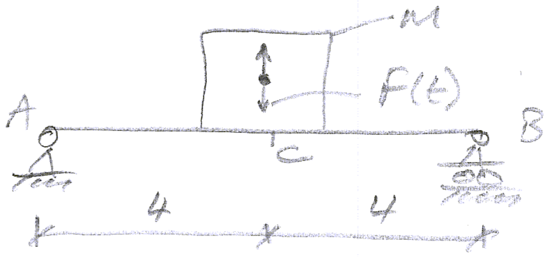
The DAF is, $D = \left[(1-\beta^2)^2 + (2\xi\beta)^2\right]^{-1/2} = 1.372$

$$\therefore \quad u = u_{st} \times 1.372 = \underline{2.92\text{mm}}$$

b) When $\beta = 1$, $D = 1/2\xi = 1/2\times0.04 = 12.5$

$$\therefore \quad u_{\beta=1} = 2.125\times12.5 = \underline{26.56\text{mm}}$$

Example 2.8



An air-conditioning unit sits on a beam as shown. The motor runs at 300 rpm and produces an unbalanced load of 120 kg. Assuming a damping ratio of 5% and neglecting the weight of the beam, determine the steady-state amplitude & deflection at C. What rpm will result in resonance and what is the associated deflection?

$EI = 8\times10^3\ kNm^2$

$m = 1{,}600\ kg$

$F_0 = 120\times9.81/10^3 = 1.18\ kN$

$\Omega = 300/60\times2\pi = 10\pi\ rad/s$

$\left.\right\} F(t) = 1.18\sin 31.41t$

$\delta_c = PL^3/48EI \therefore \frac{P}{\delta} = 48\frac{EI}{L^3} = k = \frac{48\times8\times10^3}{8^3} = 750\ kN/m$

The static deflection is, $u_{st} = \frac{1600\times9.81/10^3}{750} = 20.93\,mm$

$\omega = \sqrt{k/m} = \sqrt{\frac{750\times10^3}{1600}} = 21.65\ rad/s \quad \therefore f = \frac{\omega}{2\pi} = 3.45\,Hz$

$\beta = \frac{\Omega}{\omega} = 10\pi/21.65 = 1.451$. With $\xi = 0.05$, the DAF is

$\therefore D = \left[(1-\beta^2)^2 + (2\xi\beta)^2\right]^{-1/2} = 0.89676$

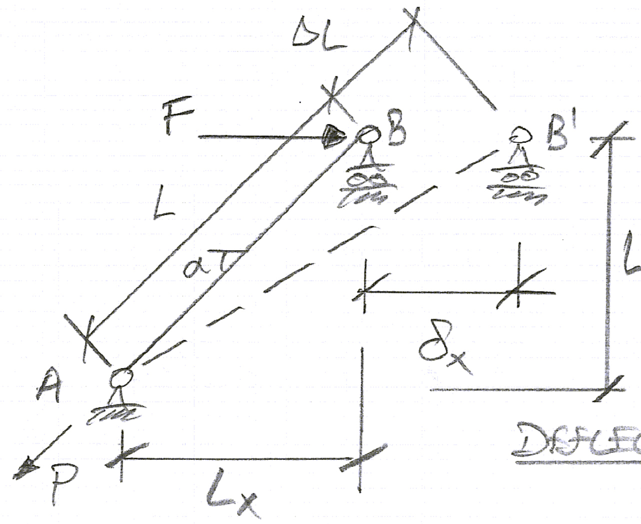
$\therefore e_{max} = 0.89676\times\frac{1.18}{750}\times10^3 = 1.41\,mm$

$\therefore \delta_c = 20.93 + 1.41 = \underline{22.34\,mm}$

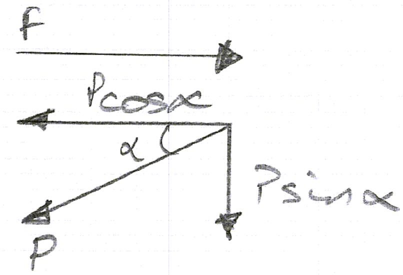
$\beta = 1;\ \Omega = \omega \quad \therefore \Omega_{rpm} = \frac{60}{2\pi}\times21.65 = \underline{206.74\,rpm}$

$D_{\beta=1} = 1/2\xi = 10 \therefore \delta_c = 20.93 + 10\left[\frac{1.18}{750}\times10^3\right] = \underline{36.66\,mm}$

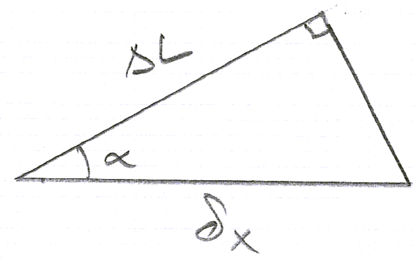
HORIZONTAL STIFFNESS OF AN INCLINED MEMBER



FORCES:



DEFLECTIONS:



NOTING,

$$\Delta L = \frac{PL}{EA}$$

BUT, FROM THE DEFLECTIONS,

$$\cos\alpha = \frac{\Delta L}{\delta_x} \quad \therefore \quad \Delta L = \delta_x \cos\alpha$$

AND FORCES:

$$F = P\cos\alpha \quad \therefore \quad P = F/\cos\alpha$$

$$\therefore \quad \delta_x \cos\alpha = \frac{F/\cos\alpha \; L}{EA}$$

$$\therefore \quad \delta_x \cdot \frac{EA}{L} \cdot \cos^2\alpha = F$$

$$\therefore \quad K_H = \frac{F}{\delta_x} = \frac{EA}{L}\cos^2\alpha$$

$$= EA \cdot \frac{L_x^2}{(L_x^2 + L_y^2)^{1.5}}$$