6.3 Amplitude Solution to Equation of Motion

The solution to the equation of motion is found to be in the form:

$$u(t) = A\cos\omega t + B\sin\omega t \tag{6.1}$$

However, we regularly wish to express it in one of the following forms:

$$u(t) = C\cos(\omega t + \alpha) \tag{6.2}$$

$$u(t) = C\cos(\omega t - \beta) \tag{6.3}$$

Where

$$C = \sqrt{A^2 + B^2} \tag{6.4}$$

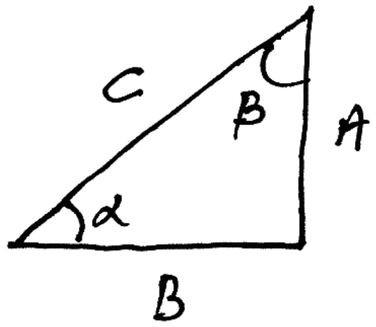
$$\tan\alpha = \frac{A}{B} \tag{6.5}$$

$$\tan\beta = \frac{B}{A} \tag{6.6}$$

To arrive at this result, re-write equation (6.1) as:

$$u(t) = C\left[\frac{A}{C}\cos\omega t + \frac{B}{C}\sin\omega t\right] \tag{6.7}$$

If we consider that *A*, *B* and *C* represent a right-angled triangle with angles α and β, then we can draw the following:



Thus:

$$\sin\alpha = \cos\beta = \frac{A}{C} \tag{6.8}$$

$$\cos\alpha = \sin\beta = \frac{B}{C} \tag{6.9}$$

Introducing these into equation (6.7) gives two relationships:

$$u(t) = C\left[\sin\alpha\cos\omega t + \cos\alpha\sin\omega t\right] \tag{6.10}$$

$$u(t) = C\left[\cos\beta\cos\omega t + \sin\beta\sin\omega t\right] \tag{6.11}$$

And using the well-known trigonometric identities:

$$\sin(X+Y) = \sin X\cos Y + \cos X\sin Y \tag{6.12}$$

$$\cos(X-Y) = \cos X\cos Y + \sin X\sin Y \tag{6.13}$$

Gives the two possible representations, the last of which is the one we adopt:

$$u(t) = C\sin(\omega t + \alpha) \tag{6.14}$$

$$u(t) = C\cos(\omega t - \beta) \tag{6.15}$$